STATISTICAL HYPOTHESES – SOLUTIONS

1. A random sample of 100 recorded deaths in the United States during one of the past years showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the average life span today is greater than 70 years? Use a $(1 - 0.05)$ confidence level (termed also 0.05 **significance** level).
Hint. In other words — $H0: \ \mu = 70$ years and $H1: \ \mu > 70$ years.
The critical region for such $H1$ is the right-hand tail of the standard normal distribution – why? $(u_{0.95}, \infty) = (1.645, \infty)$. *The calculated standardized value $X = 71.8$ converted into standard Gaussian) is 2.02. $H0$ must be rejected in favor of $H1$.*

2. The Edison Electric Institute has published figures on the annual number of kilowatt-hours expended by various home appliances. It is claimed that a vacuum cleaner expends an average of 46 kW-hrs per year. A random sample of 12 homes included in a planned study indicates that vacuum cleaners expend an average of 42 kW-hrs (sample arithmetic average) per year and the sample unbiased estimator (S^*) of the standard deviation is 11.9 kW-hrs. Does it suggest at $\alpha = 0.05$ significance level that vacuum cleaners expend, on the average, less than 46kW-hrs annually?
Hint. In other words — $H0: \ \mu = 46$ kW-hrs and $H1: \ \mu < 46$ kW-hrs. Since we do not know σ we have to use its estimate (S^*) and, consequently, the Gaussian curve must be replaced by the Student's one. The critical region for this $H1$ hypothesis is the left-hand tail of the Student's distribution – why? $(-\infty, t_\alpha) = (-\infty, -1.796)$. *The calculated standardized value ($X = 46$ converted into Student t) is -1.16. Decision ?*

3. Two detergents are tested for their efficiency when used for washing woolen fabrics. For the detergent A 10 test results (roughly: the percentages of the properly washed area of fabric) are: 74.8, 75.1, 73, 72.8, 76.2, 74.6, 76, 73.4 72.9, and 71.6. For the detergent B we have 7 results: 56.9, 57.8, 54.6, 59, 57.1, 58.2, and 57.6. Can we say — using $\alpha = 0.05$ — that detergent A is better than B?

   Hint. This time — $H0: \ \mu_A = \mu_B$ against and $H1: \ \mu_A > \mu_B$ (such $H1$ is the only logical one, right? This requires some work: first we must calculate $\bar{x}_A$, $\bar{x}_B$, ${S^*}^2_A$, ${S^*}^2_B$ (74, 57.3, 2.31 and 1.92, respectively); then we must verify the hypothesis about $\sigma_A = \sigma_B = \sigma$ (unknown) using the Fisher test. We have $\mathcal{F} = 1.20$ — a value that is far away from the critical $[??, \infty)$ region. Finally the Δ value (cf. lecture) must be calculated and confronted with the appropriate Student quantile $t(0.95, 15) = 1.75$. The A detergent <u>is</u> a better one.

4. Suppose you are to decide which of two equally-priced brands of light bulbs lasts longer. You choose a random sample of 100 bulbs of each brand and find that brand $\mathcal{A}$ has sample mean $\bar{X}_A$ of 1 180 hours and sample standard deviation σ_A of 120 hours, and that brand $\mathcal{B}$ has sample mean $\bar{X}_B$ of 1 160 hours and sample standard deviation σ_A of 40 hours. What decision should you make at 5% significance level?

   Hint: Because $n_A + n_B = 200$ is greater than 30 we may assume that the standardized difference Z follows a normal distribution:
$$Z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_A - \mu_B)}{\sqrt{\sigma_{\bar{X}_A - \bar{X}_B}}} = ?$$
$$Z = \frac{(\bar{X}_A - \bar{X}_B) - (\mu_A - \mu_2 B)}{\sqrt{\dfrac{\sigma_A^2}{n_A} + \dfrac{\sigma_B^2}{n_B}}} = \ldots \quad \text{by H0} \quad \ldots \frac{\bar{X}_A - \bar{X}_B}{\sqrt{\dfrac{\sigma_A^2}{n_A} + \dfrac{\sigma_B^2}{n_B}}}$$
   In our case $Z = 1.58$ – it's inside the non-rejection region $|Z| < 1.96$.

5. Suppose we have a type of battery for which we take a sample of $n_1 = 10$ batteries. The mean operating life of these batteries is $\bar{X}_1 = 18.0$ hours with a standard deviation, taken as $S_1 = 3.0$ hours. Suppose also that we have a new type of battery for which we take a sample of $n_2 = 17$ batteries. The mean of this sample is $\bar{X}_2 = 22.0$ hours with a standard deviation, taken as $S_2 = 3.0$ hours.

   Determine for a 1% level of significance whether there is **a significant difference between the means of the two samples.**

Also determine for a 1% level of significance whether we can conclude that **the new batteries are superior to the old ones.**

First question: $H0: \quad \mu_1 = \mu_2; \qquad H1: \quad \mu_1 \neq \mu_2.$

$$\Delta \;=\; \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{n_1 S_1^2 + n_2 S_2^2}} \times \sqrt{\frac{n_1 n_2 (n_1 + n_2 - 2)}{n_1 + n_2}}$$

follows Student's distribution with $n_1 + n_2 - 2$ degrees of freedom.

For $H0$ hypothesis the rejection region would be $\Delta \leq -2.787 \bigcap \Delta \geq 2.787$; Here we have $\Delta = -1.96$ – we have no reason to reject $H0$.

However to answer the second question the rejection region is: $\Delta \leq -2.485$; still the second $H0$ may be not rejected.