NORMAL DISTRIBUTION...

...is by far the most important one. Therefore, we must gain some basic abilities to use the numerical data pertaining to this chapter (important!) of statistics.
Notation: μ is the expected value, i.e. $E(X)$. σ – the mean standard deviation: $\sqrt{\mathrm{VAR}(X)}$.

With the help of the table of areas under the normal curve (i.e. - values of the cumulative distribution; you may also use dedicated software – e.g. Excel) solve the following simple problems:

- Given a standard normal distribution, find the area under the curve that lies (a) to the right of $z = 1.84$, and (b) between $z = 1.97$ and $z = 0.86$.
- Given a standard normal distribution, find the value of k such that (a) $P(Z > k) = 0.3015$. and (b) $P(k < Z < -0.18) = 0.4197$.
- Given a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62. Hint: you must *standardise* our RV; $X \rightarrow Z = \dfrac{X - \mu}{\sigma}$.
- Given a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.
- Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has (a) 45% of the area to the left, and (b) 14% of the area to the right.
- A certain type of storage battery lasts on the average 3.0 years; with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.
- An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.
- In an industrial process the diameter of a ball bearing must meet the specification: 3.0 ± 0.01 cm. It is known that in the process the diameter of the ball bearing has a normal distribution with mean 3.0 and standard deviation $\sigma = 0.005$. On the average; how many ball bearings will be scrapped?
- On an examination the average grade was 74 and the standard deviation was 7. If 12% of the class are given A's; and the grades follow a normal distribution what is the lowest possible A and the highest possible B?
- For the problem above find the sixth *decile*, i.e. the $q_{0.6}$ quantile. (This is the x-value that leaves 60% of the area under the normal curve to the left.)

Exponential probability density function

- Suppose the length of time an electric bulb lasts, X, is a random variable with cumulative function

$$F(x) = \mathcal{P}(X \leq x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/500} & x \geq 0. \end{cases}$$

  Find the probability that the bulb lasts: (a) between 100 and 200 hours; (b) beyond 300 hours; (c) Find the expected value of the bulb life-time

- Let X be the random variable representing the length of a telephone conversation. Let $f(x) = \lambda e^{-\lambda x}, \quad 0 \leq x < \infty$. Find the c.d.f $F(x)$ and find $\mathcal{P}(5 < X < 10)$.