(2) – CONDITIONAL PROBABILITY; BAYES THEOREM

1. A - defective using the 1st way;
   B - defective using the 2nd way;
   (a) $\mathcal{P}(\bar{A}) = 0.95 \cdot 0.9 \cdot 0.7 = 0.599; \qquad \mathcal{P}(\mathcal{A}) = 0.401$
   $\mathcal{P}(\bar{B}) = 0.75^2 = 0.56; \qquad \mathcal{P}(B) = 0.44$
   $\mathcal{P}(A) < \mathcal{P}(B) \rightarrow$ 1st way better
   (b) $\mathcal{P}(\mathit{defective}) = \mathcal{P}(1st)\mathcal{P}(A) + \mathcal{P}(2nd)\mathcal{P}(B) = 0.5\mathcal{P}(A) + 0.5\mathcal{P}(B) = 0.58$
2. (a) $\mathcal{P}(\mathit{defective}) = \sum_i \mathcal{P}(\mathit{plant}_i)\mathcal{P}(\mathit{defective}|\mathit{plant}_i) = 0.0108$
   (b) similar formula $\mathcal{P} = 0.107$
   (c) $\mathcal{P} = \mathcal{P}(1st)\mathcal{P}(2nd) + \mathcal{P}(2nd)\mathcal{P}(1st) = 0.17$
3. $\mathcal{P}(7)$ in the 1st throw $= 1/6$; $\mathcal{P}(11$ in the 2nd $= 1/18$ or vice versa $= 1/54$
4. dependent $\mathcal{P}(A \cap B) = 0.04 \neq \mathcal{P}(A) \cdot \mathcal{P}(B)$
   $\mathcal{P}(A|B) = \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(B)} = 0.5$
5. You should consider the 6×6 diagram; BUT only eleven out of 36 cases have (at least one) 4. These are
   $(4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (1,4), (2,4), (3,4), (5,4), (6,4)$. So $\Omega = 11$.
   Then $\mathcal{P}(a) = 2/11$ (5 appears only twice); $\mathcal{P}(b) = 5/11$.
6. $\mathcal{P}(\mathit{not\ documented}) = (0.4)^3 \rightarrow \mathcal{P}(\mathit{documented}) = 1 - 0.4^3 = 0.936$

$$1 - 0.4^n \geq 0.99; \qquad n \geq \ln 40$$

7. $\mathcal{P}(\mathit{sick}|\mathit{test\ \ positive}) = \frac{\mathcal{P}(\mathit{test\ \ positive}|\mathit{sick}) \times \mathcal{P}(\mathit{sick})}{\mathcal{P}(\mathit{test\ \ positive})} \approx 0.17$
8. $\mathcal{P}(\mathit{taxi\ being\ green}) = 0.41 < 0.5$!
9. First •

$$\mathcal{P}(F_{AB}|Ch_{OA})\mathcal{P}(Ch_{OA}) = \mathcal{P}(Ch_{OA}|F_{AB})\mathcal{P}(F_{AB}) \quad \text{thus we have}$$

$$\mathcal{P}(F_{AB}|Ch_{OA}) = \frac{\mathcal{P}(Ch_{OA}|F_{AB})\mathcal{P}(F_{AB})}{\mathcal{P}(Ch_{OA})} = \ldots$$

$$\mathcal{P}(Ch_{OA}) = \mathcal{P}(Ch_{OA}|F_{OA})\mathcal{P}(F_{OA}) + \mathcal{P}(Ch_{OA}|F_{AA})\mathcal{P}(F_{AA}) + \mathcal{P}(Ch_{OA}|F_{AB})\mathcal{P}(F_{AB}) = \ldots 0,2525$$

$$\ldots = \frac{0.5 \times 0.15}{0,2525} \approx 0,297.$$

10. Second •

    the portions of unit square *unfavorable* for meeting to take place are two right isosceles triangles in the upper-left and bottom-right corners of the square; the side of each triangle is 3/4 hour. So the total unfavorable area is $2 \times 3/4 \times 3/4 \times 1/2 = 9/16$. The favorable area $= 7/16$