PROBABILITY – BASIC QUESTIONS

1. A poker hand consists of 5 cards. Find the probability of holding 2 kings and 1 queen.

$$\mathcal{P} = \frac{\binom{4}{2}\binom{4}{1}\binom{44}{2}}{\binom{52}{5}}.$$

2. The probabilities that a person selecting a new car will choose the colour green, white, red or blue are 0.09, 0.15, 0.21 and 0.23, respectively. What the probability that a given car will purchase a car that comes in blue or green?

$$\mathcal{P} = 0.23 + 0.09$$

3. A box contains 500 envelopes of which 75 contain $ 100 in cash, 150 – $ 25, and 275 contain $ 10. Find the probability that an envelope selected at random contains less than $ 100.

$$\mathcal{P} = (150 + 275)/500 = 1 - 75/500$$

4. What is the probability of getting total of 7 or 11 when a pair of dice are tossed?

   Hint: construct a 6x6 matrix and count carefully the number of realisations of the sum equal to 7 AND 11. Than divide this number by 36.

5. How many even, 4-digit numbers can be constructed, assuming zero cannot appear in the first position ? Consider 2 cases : (a) all digits must be different ; (b) each digit can be used any number of times.
   Hint: *remember about rejecting the* $0xxx$ *numbers; the first digit can be chosen in 8 or 9 (a) or 9(b) manners.*

   (a)
   5 possibilities for the last digit and $P_{9,3}$ for the first three; but this includes the numbers with 0 in the 1st position. There is $4 \cdot P_{8,2}$ of them. After substraction 2296
   (b)

$$9 \times 10 \times 10 \times 5 = 4500$$

6. 3 boys and 4 girls walk in Indian file. Calculate the number of the file arrangements if any two closest members of the file are to be of opposite sex.
   Hint: *the file must start with a girl. Permutations.*

$$4! \times 3! = 144$$

7. N persons have been invited to a party ($N \leq 12$). Assuming that the probability of being born under one of 12 Zodiac signs is the same (not true, actually) calculate the probability of : (a) at least 2 persons being of the same sign ; (b) all persons being of the same sign.

Hint:*(a) calculate the P of all persons being of different signs ; (b) easy —if you know already the number of all events in the event space.*

cf. the Feller problems in the lecture
`http://www.fis.agh.edu.pl/~lenda/stat/wykl_02_probabil.pdf`

8. The total number of fish in a pool is N. Among them, k have been marked with a (non-washable, ecological) paint. We fish out n fish. What is the P that none of them is marked ? Consider two scenarios : (a) we keep every caught fish out of the pool ; (b) every fished-out fish is returned to the pool before the next try.
(a)Number of possible scenarios is combination $C_{N,n} = \binom{N}{n}$.
For UNMARKED $C_{N-k,n} = \binom{N-k}{n}$. The probability is the ratio of the two.
(b)
$$\mathcal{P} = \left(\frac{N-k}{N}\right)^n .$$