Time Crystals

Krzysztof Sacha

Jagiellonian University in Kraków.

Outline:

- Formation of crystals:
  - Space crystals
  - Time crystals
- Modeling time crystals:
  - Anderson localization in the time domain (D. Delande, K. Giergiel, L. Morales-Molina)
  - Mot insulator phase in the time domain

Formation of crystals

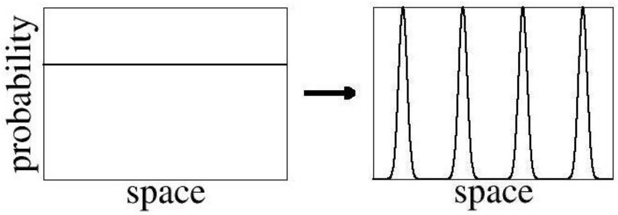
Formation of space crystals

$$[\hat{H}, \hat{T}] = 0$$

$\hat{H}$ – solid state system Hamiltonian
$\hat{T}$ – translation operator of all particles by the same vector

$$\left|\hat{T}\psi\right|^2 = \left|e^{i\alpha}\psi\right|^2 = |\psi|^2$$

t =const.

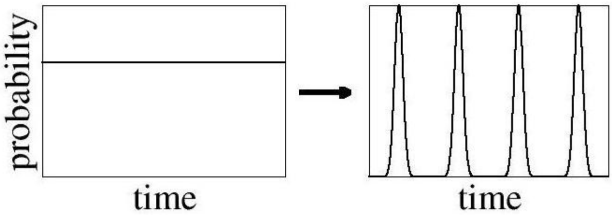


Formation of time crystals?

Eigenstates of a time-independent Hamiltonian H are also eigenstates of a time translation operator e^{-iHt}

$$\left|e^{-iHt}\psi\right|^2 = \left|e^{-iEt}\psi\right|^2 = |\psi|^2$$

$\vec{r}$ is fixed



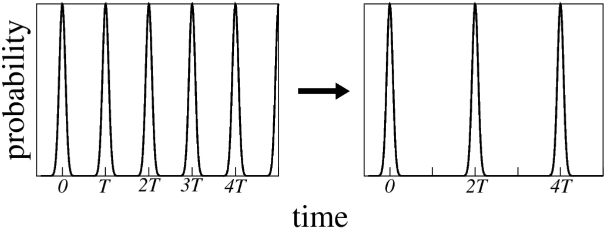
F. Wilczek, PRL **109**, 160401 (2012).
T. Li *et al.*, PRL **109**, 163001 (2012).
J. Zakrzewski, Physics **5**, 116 (2012).
P. Coleman, Nature **493**, 166 (2013).
KS, PRA **91**, 033617 (2015).

P. Bruno, PRL **110**, 118901 (2013).
F. Wilczek, PRL **110**, 118902 (2013).
P. Bruno, PRL **111**, 029301 (2013).
T. Li *et al.*, arXiv:1212.6959.
P. Bruno, PRL **111**, 070402 (2013).

A. Syrwid, J. Zakrzewski, KS, "Time crystal behavior of excited eigenstates", arXiv:1702.05006

Discrete time crystals

Spontaneous process



Discrete time crystals

Single particle systems

Example: single particle bouncing on an oscillating mirror in 1D:

A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. **368**, 409 (2002).

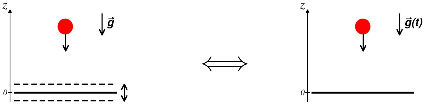
Floquet Hamiltonian

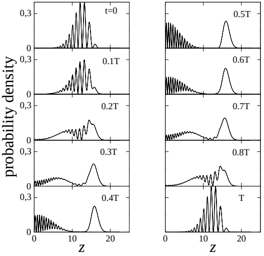
$$H_F(t) = -\frac{1}{2}\frac{\partial^2}{\partial z^2} + z + \lambda z \cos(2\pi t/T) - i\frac{\partial}{\partial t}$$

$$H_F\ \psi_n(z,t) = E_n\ \psi_n(z,t)$$

E_n – quasi-energy

$\psi_n(z,t)$ – time periodic function





2 : 1 resonance

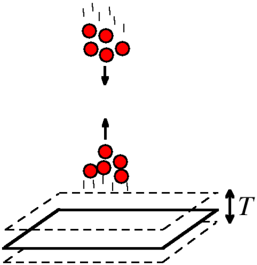
$\lambda = 0.06,\ T = 5.7$

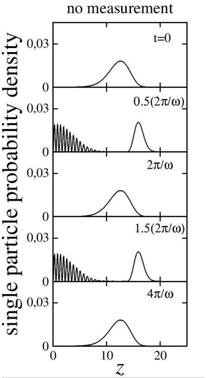
Discrete time crystals

Bosons with attractive interactions

Two-mode approximation: $\hat{\psi} \approx \phi_1(z,t)\,\hat{a}_1 + \phi_2(z,t)\,\hat{a}_2,$

$g_0 N = -0.5, \quad N = 10^4$





$|\psi\rangle \approx |N,0\rangle + |0,N\rangle$

KS, Phys. Rev. A **91**, 033617 (2015).

Discrete time crystals

Bosons with attractive interactions

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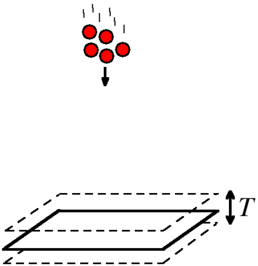
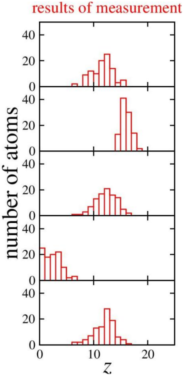




$|\psi\rangle \approx |N,0\rangle + |0,N\rangle \quad \longrightarrow \quad |\psi\rangle \approx |N-1,0\rangle$ or $|0,N-1\rangle$

KS, Phys. Rev. A **91**, 033617 (2015).

Discrete time crystals

Experiments

V. Khemani, A. Lazarides, R. Moessner, L. S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

D. V. Else, B. Bauer, C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).

LETTER

doi:10.1038/nature21413

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

LETTER

doi:10.1038/nature21426

Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

Discrete time crystals

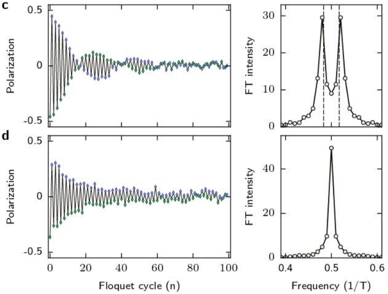
Experiments

Chain of 10 ions (spins):

$$H = \begin{cases} H_1 = g(1-\varepsilon)\sum_i \sigma_i^y, & \text{time } t_1 \\ H_2 = \sum_i J_{ij}\sigma_i^x\sigma_j^x, & \text{time } t_2 \\ H_3 = \sum_i D_i\sigma_i^x & \text{time } t_3. \end{cases}$$

J. Zhang *et al.*, Nature (2017).

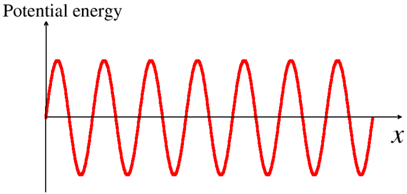
10^6 impurities in diamond:



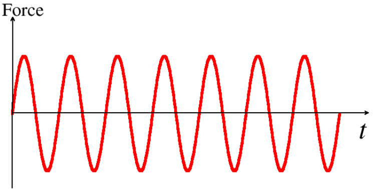
S. Choi *et al.*, Nature (2017).

Modeling time crystals

Modeling space crystals: $H(x+\lambda) = H(x)$



Modeling time crystals: $H(t+T) = H(t)$



Modeling time crystals

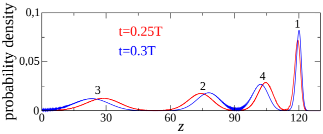
Single particle systems

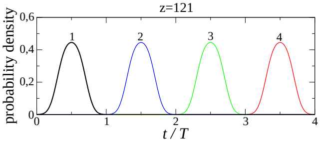
In the $s : 1$ resonance case:

- There are s Floquet eigenstates with quasi-energies $E_j \approx E_{j'}$.
  These quasi-energies form a band when $s \to \infty$.

- s individual wavepackets, $\phi_j(z,t)$, can be prepared by superposing s Floquet eigenstates.
  For $s \to \infty$, the wavepackets $\phi_j(z,t)$ become analogues of Wannier states but in the time domain.

Example for $s = 4$:





Restricting to the Hilbert subspace $\psi = \sum_{j=1}^{s} a_j \phi_j$,

$$E_F = \int_0^{sT} dt \langle \psi | H_F | \psi \rangle \approx -\frac{J}{2} \sum_{j=1}^{s} (a_{j+1}^* a_j + \text{c.c.})$$

$$J = -2 \int_0^{sT} dt \langle \phi_{j+1} | H_F | \phi_j \rangle$$

The lowest and higher quasi-energy bands can be considered.

KS, Sci. Rep. **5**, 10787 (2015).

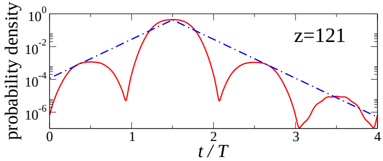
Anderson localization in the time domain

$$E_F = -\frac{J}{2}\sum_{j=1}^{s}(a_{j+1}^{*}a_j + \text{c.c.}) + \sum_{j=1}^{s}\varepsilon_j |a_j|^2$$

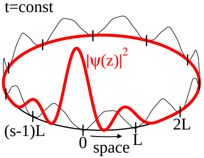
with $\varepsilon_j = \int_0^{sT} dt \langle \phi_j | H'(t) | \phi_j \rangle$,

where $H'(t)$ is a perturbation that fluctuates in time but $H'(t + sT) = H'(t)$.

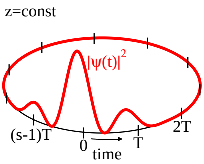
Example for $s = 4$:



Space Crystal



Time Crystal

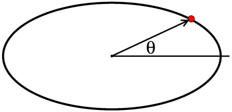


KS, Sci. Rep. **5**, 10787 (2015).

Anderson localization in the time domain

- Anderson localization in time without non-spreading wave-packets:

$$H = \frac{p^2}{2} + V\, g(\theta)\, f(t),$$



$g(\theta)$ is a regular function, $f(t)$ fluctuates randomly.

KS, D. Delande, Phys. Rev. A **94**, 023633 (2016).

- Anderson localization of an electron along a Kepler orbit in an Hydrogen atom perturbed by a fluctuating microwave field.

K. Giergiel, KS, *in preparation*.

- Phase transition in Anderson localization in the time domain — time crystals with properties of 3D systems.

D. Delande, L. Morales-Molina, KS, arXiv:1702.03591.

Mott insulator-like phase in the time domain

Many-body systems

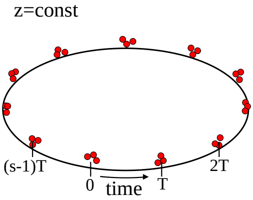
Bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}^\dagger_{j+1}\hat{a}_j + \mathrm{h.c.}) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\hat{n}_i\hat{n}_j$$

with $U_{ij} = g_0 \int_0^{sT} dt \int_0^{\infty} dz |\phi_i|^2\ |\phi_j|^2$, where $|U_{ii}| > |U_{ij}|$ for $i \neq j$.

- For $g_0 \to 0$, the ground state is a superfluid state with long-time phase coherence.
- For strong repulsion, $U_{ii} \gg NJ/s$, the ground state becomes a Fock state $|N/s, N/s, \ldots, N/s\rangle$ and long-time phase coherence is lost.

Time Crystal



KS, Sci. Rep. **5**, 10787 (2015).

Summary:

- Time crystals are analogues of space crystals but in the time domain.
- We have shown that spontaneous breaking of discrete time translational symmetry is possible.
  Recently it has been demonstrated experimentally.
- Space periodic potentials are often used to model properties of space crystals.
  Crystalline structures in the time domain can be modeled by periodically driven systems.
- We show that Anderson localization and Mott insulator-like phase can be observed in the time domain.

Recovery of Wilczek model

A. Syrwid, J. Zakrzewski, KS, arXiv:1702.05006

Bosons with attractive contact interactions on a ring:

$$H = \sum_{i=1}^{N} \frac{(p_i - \alpha)^2}{2} + \frac{g_0}{2} \sum_{i \neq j} \delta(x_i - x_j),$$

Mean field description: bright soliton solution.
The CM coordinate frame:

$$H = \frac{(P - N\alpha)^2}{2N} + \tilde{H}(\tilde{x}_i, \tilde{p}_i), \qquad P_j = 2\pi j$$

Ground state:

$$\frac{\partial H}{\partial P_j} = 2\pi \frac{j}{N} - \alpha \approx 0$$

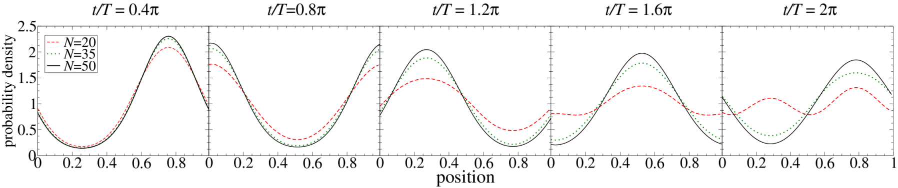
Excited state $P_N = 2\pi N$:

$$\frac{\partial H}{\partial P_N} = 2\pi - \alpha \neq 0$$

Recovery of Wilczek model

Measurement of the position x_1 of a single particle at $t = 0$ is expected to break continuous time translation symmetry:

$$\rho_2(x,t) \propto \langle\psi_0|\hat{\psi}^\dagger(x,t)\hat{\psi}(x,t)\ \hat{\psi}^\dagger(x_1,0)\hat{\psi}(x_1,0)|\psi_0\rangle,$$



If the symmetry broken state lives forever in the limit $N \to \infty$, $g_0 \to 0$ with $g_0 N = \text{const.}$, the time crystal is formed,