Dynamika multifraktali finansowych

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Department of Complex Systems Theory

Fractal Geometry of …

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractal characterization of protein contact networks

J. Plasma Physics (2014), vol. 80, part 1, pp. 43–58. © Cambridge University Press 2013 43
doi:10.1017/S0022377813000895

The MDF technique for the analysis of tokamak edge plasma fluctuations

frontiers in HUMAN NEUROSCIENCE

OPINION ARTICLE
published: 21 July 2014
doi: 10.3389/fnhum.2014.00523

Multifractal analyses of human response time: potential pitfalls in the interpretation of results

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractal detrended fluctuation analysis of particle density fluctuations in high-energy nuclear collisions

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Human Movement Science

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Multifractal formalisms of human behavior

OPEN ACCESS Freely available online PLOS ONE

Multifractal Detrended Fluctuation Analysis of Human EEG: Preliminary Investigation and Comparison with the Wavelet Transform Modulus Maxima Technique

Todd Zorick[1,3]*, Mark A. Mandelkern[2,4]

theguardian

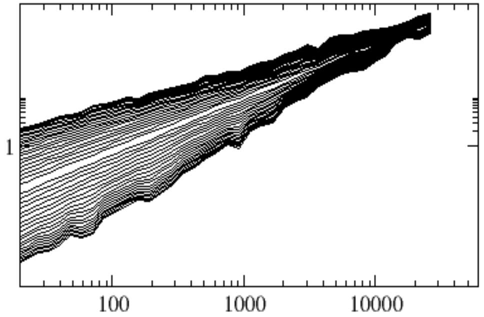
Scientists find evidence of mathematical structures in classic books

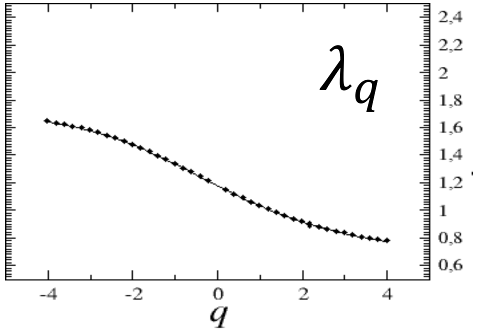
Researchers at Poland's Institute of Nuclear Physics found complex 'fractal' patterning of sentences in literature, particularly in James Joyce's Finnegans Wake, which resemble 'ideal' maths seen in nature

Multifractal methodology

Multifractal cross-correlation analysis (MFCCA)

$$F_{xy}(q,s) \sim s^{\lambda_q}$$





The q-dependent detrended cross-correlation coefficient

$$\rho_q(s) = \frac{F^q_{XY}(s)}{\sqrt{F^q_{XX}(s) F^q_{YY}(s)}}$$

The q-dependent correlation matrix

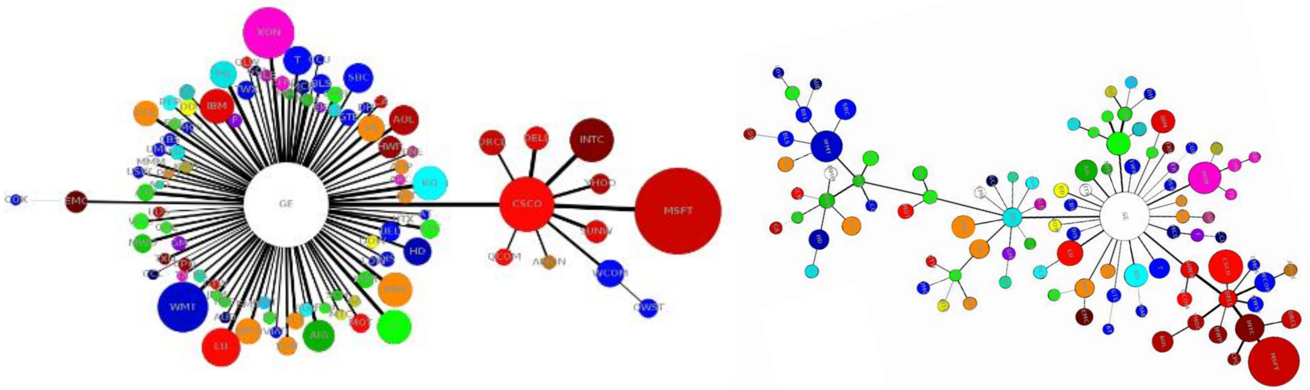
N time series

N x *N* correlation matrix C_q^s

Scale

Fluctuation amplitude selector

The q-dependent minimum spanning tree (*q*MST)



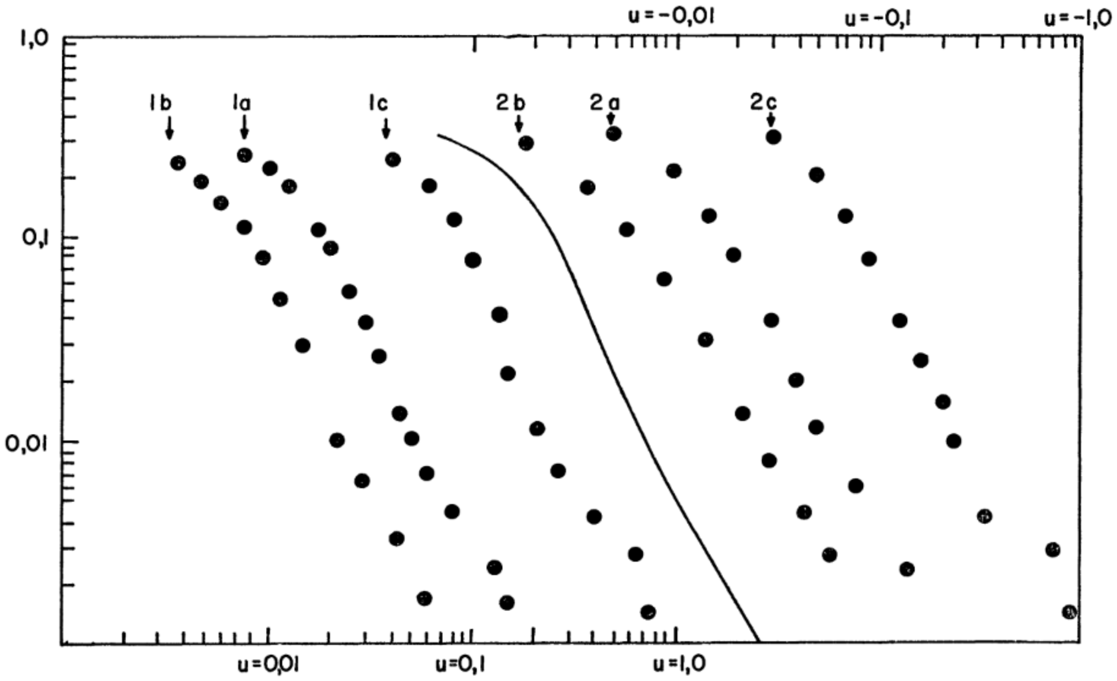
Fractal Geometry

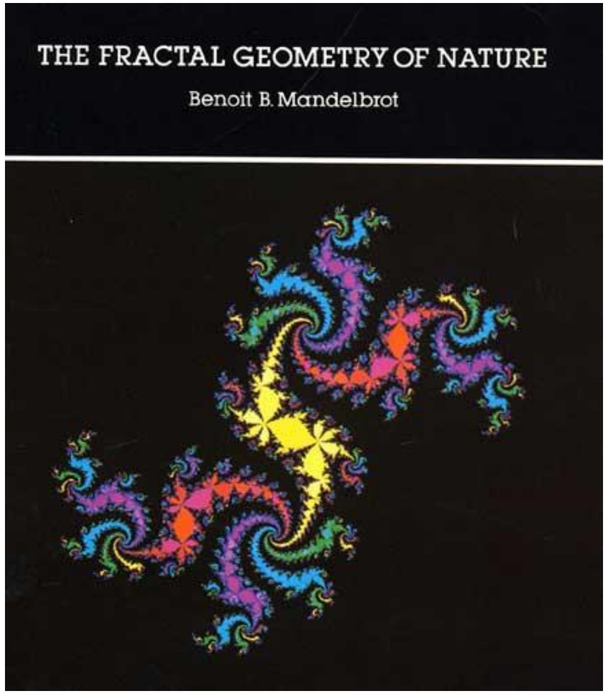
THE VARIATION OF CERTAIN SPECULATIVE PRICES*

BENOIT MANDELBROT†

The Journal of Business,

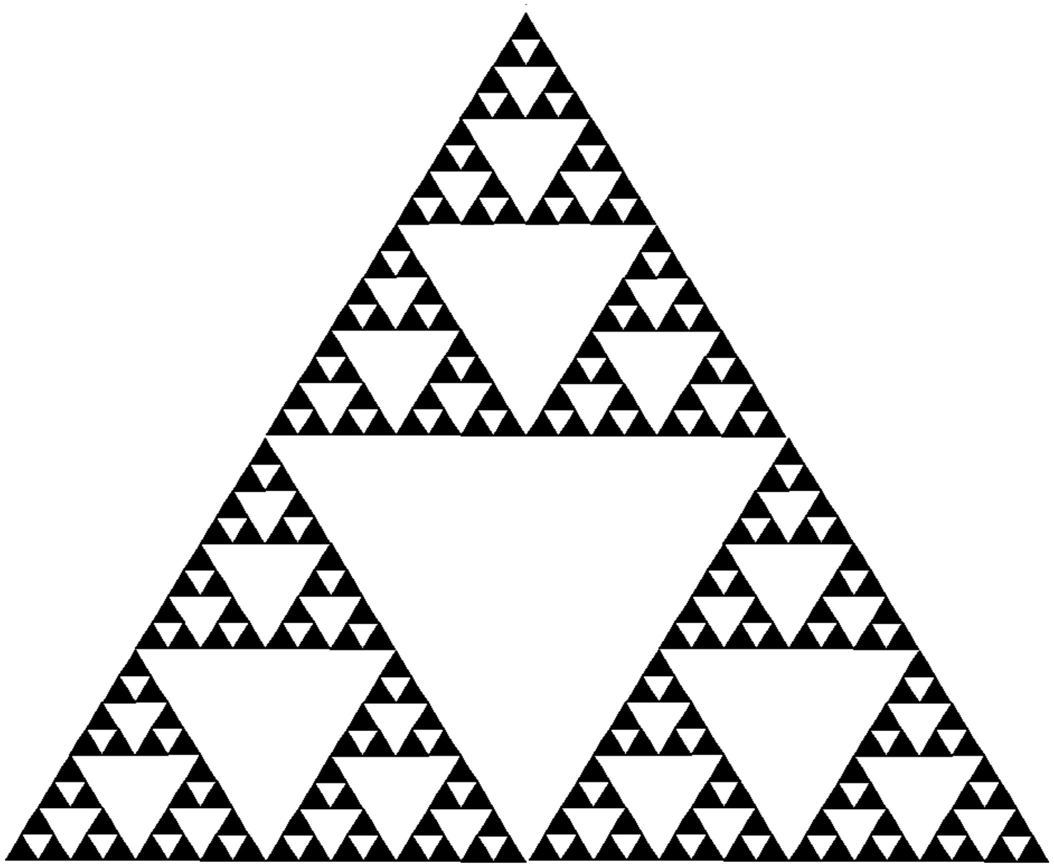
Vol. 36, No. 4 (Oct., 1963), pp. 394-419

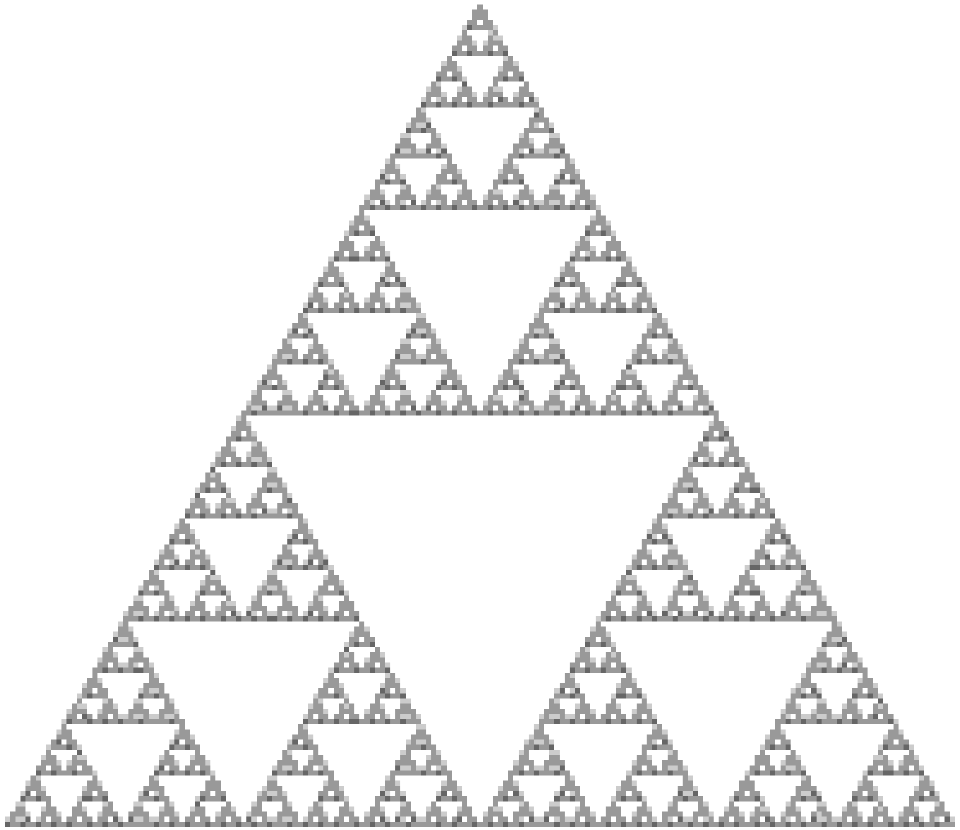




Sierpiński Triangle (1916)

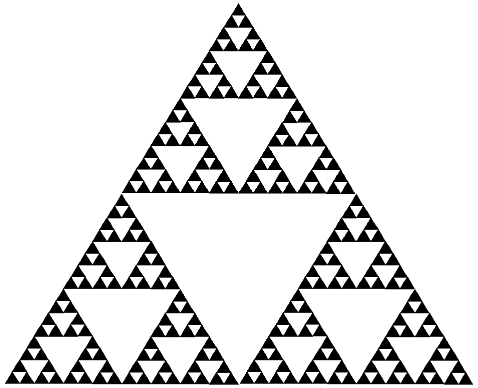
Self-similarity of the fractal structure





Fractal mosaic

Anagni Cathedral, Lazio, Italy.

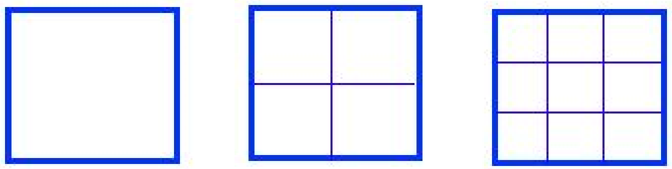


Source of pictures: internet

Fractal dimension

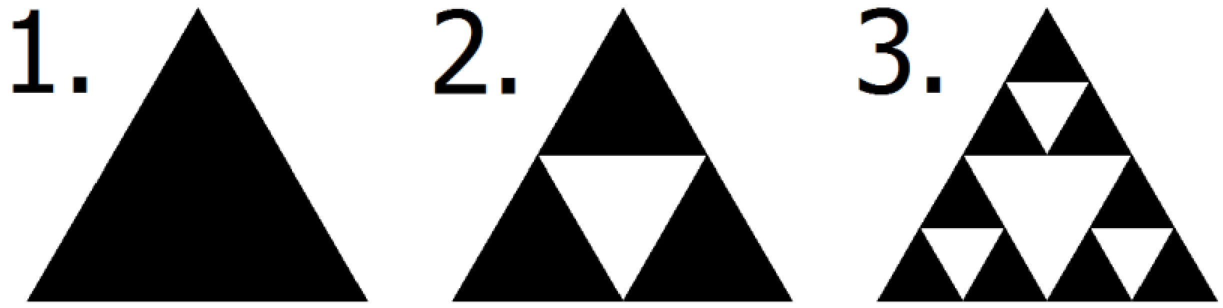
$$N \propto \varepsilon^{-d_f}$$

$$d_f = \lim_{\varepsilon \to 0} \frac{\log(N(\varepsilon))}{\log(\frac{1}{\varepsilon})}$$



$N = 1$ $N = 4$ $N = 9$

$\varepsilon = 1$ $\varepsilon = 1/2$ $\varepsilon = 1/3$



$\varepsilon=(1/2)^0=1$ $\varepsilon=(1/2)^1=1/2$ $\varepsilon=(1/2)^2=1/4$

$N=3^0=1$ $N=3^1=3$ $N=3^2=9$

$$d_f = \frac{\log(3)}{\log(2)}$$

$$d_f = 1.5849\ldots$$

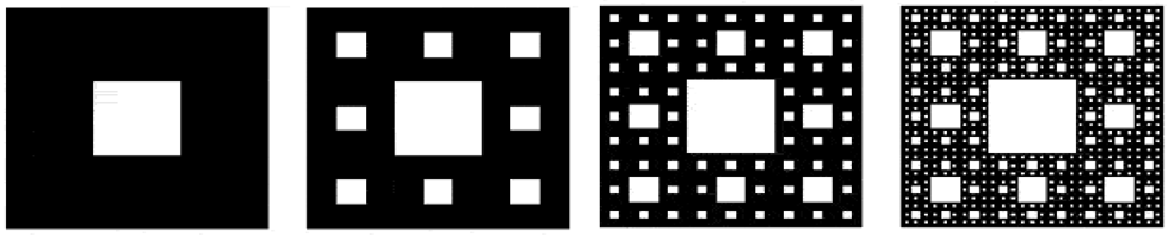
Fractal dimension

- Cantor set

d_f=0.63…

(more than point, less than segment)

- Sierpiński carpet

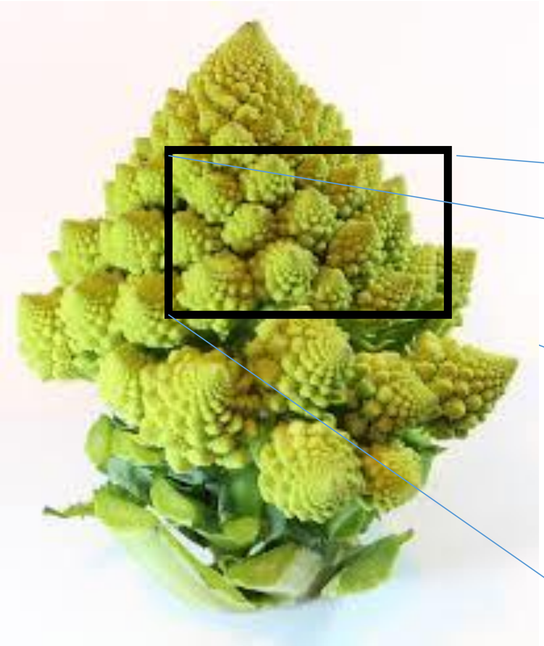


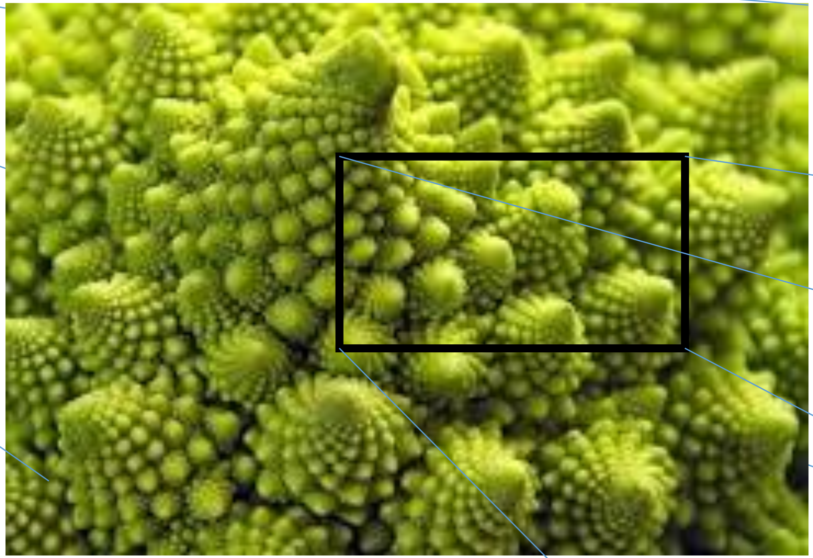
d_f=1.89…

(more than line, less than plane)

Natural Fractals

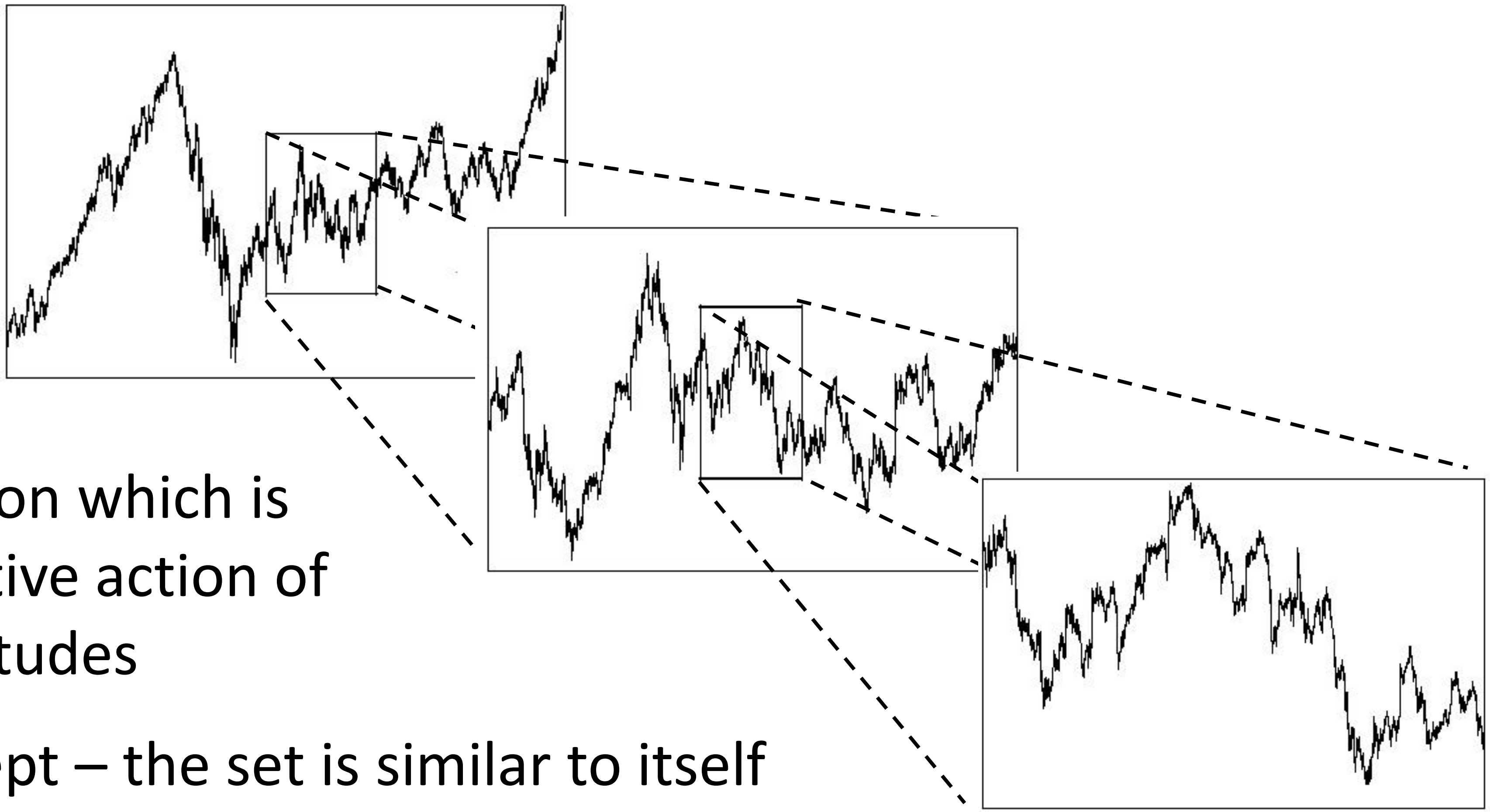
Romanesco broccoli





Source of pictures: internet

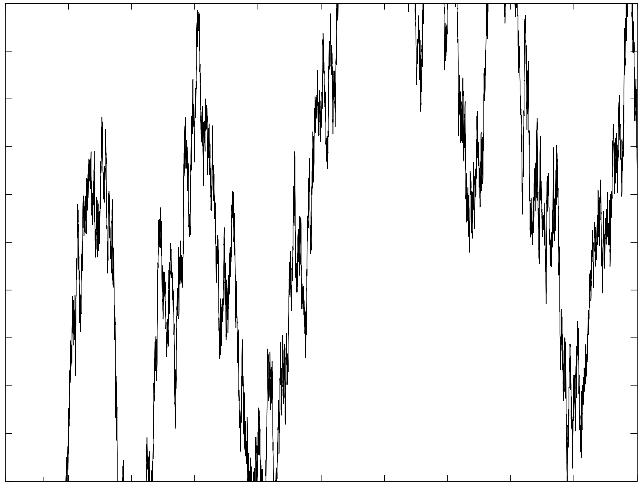
Fractal functions

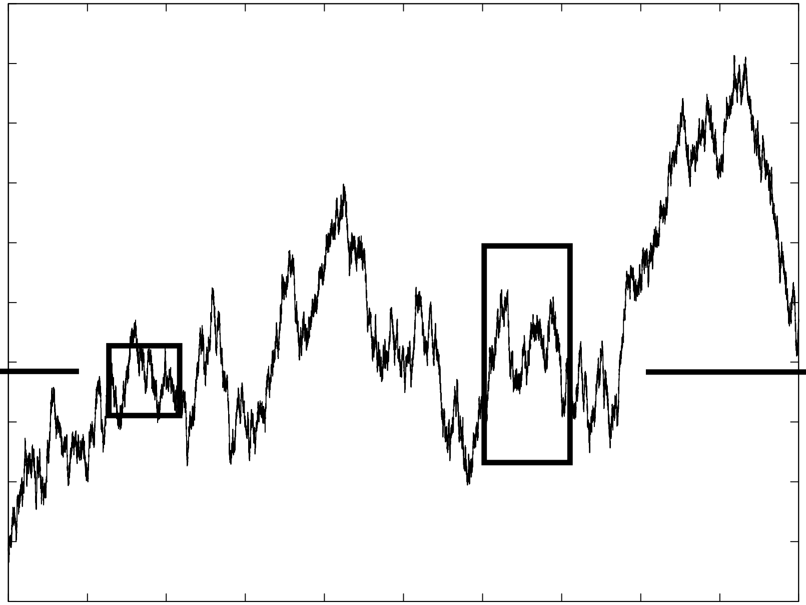


- Fractal function –
  self-similar function which is invariant by iterative action of elementary similitudes
- Self-affinity concept – the set is similar to itself when anisotropic transformation is applied

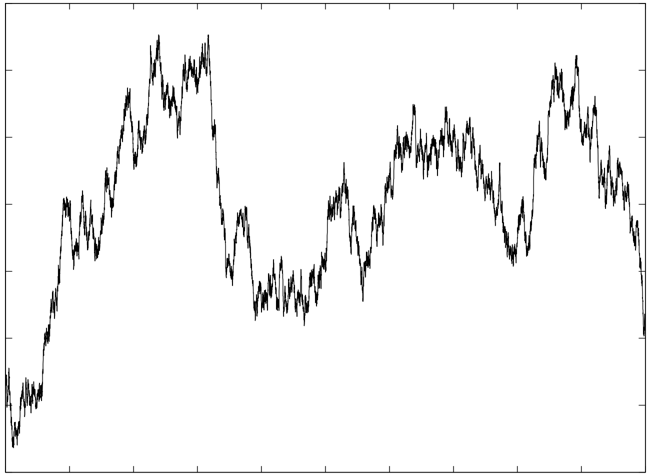
Fractal functions

Isotropic zoom





Anisotropic zoom



$$\begin{cases} x' = \lambda x \\ y' = \lambda y \end{cases}$$

$$f(x_0 + \lambda x) - f(x_0) \simeq \lambda^H (f(x_0 + x) - f(x_0))$$

$$\begin{cases} x' = \lambda x \\ y' = \lambda^H y \end{cases}$$

H is called Hurst exponent

(determines how regular the function f is)

Fractional Brownian Motion

Example of self-affine function: fractional Brownia motion (fBm)
(Mandelbrot & van Ness 1968)

$$d_f = 2 - H$$

$H \in (0,1/2)$

Antipersistent behaviour

$B_{0.2}(t)$

$d_f = 1.8$

$H = 1/2$

$B_{0.5}(t)$

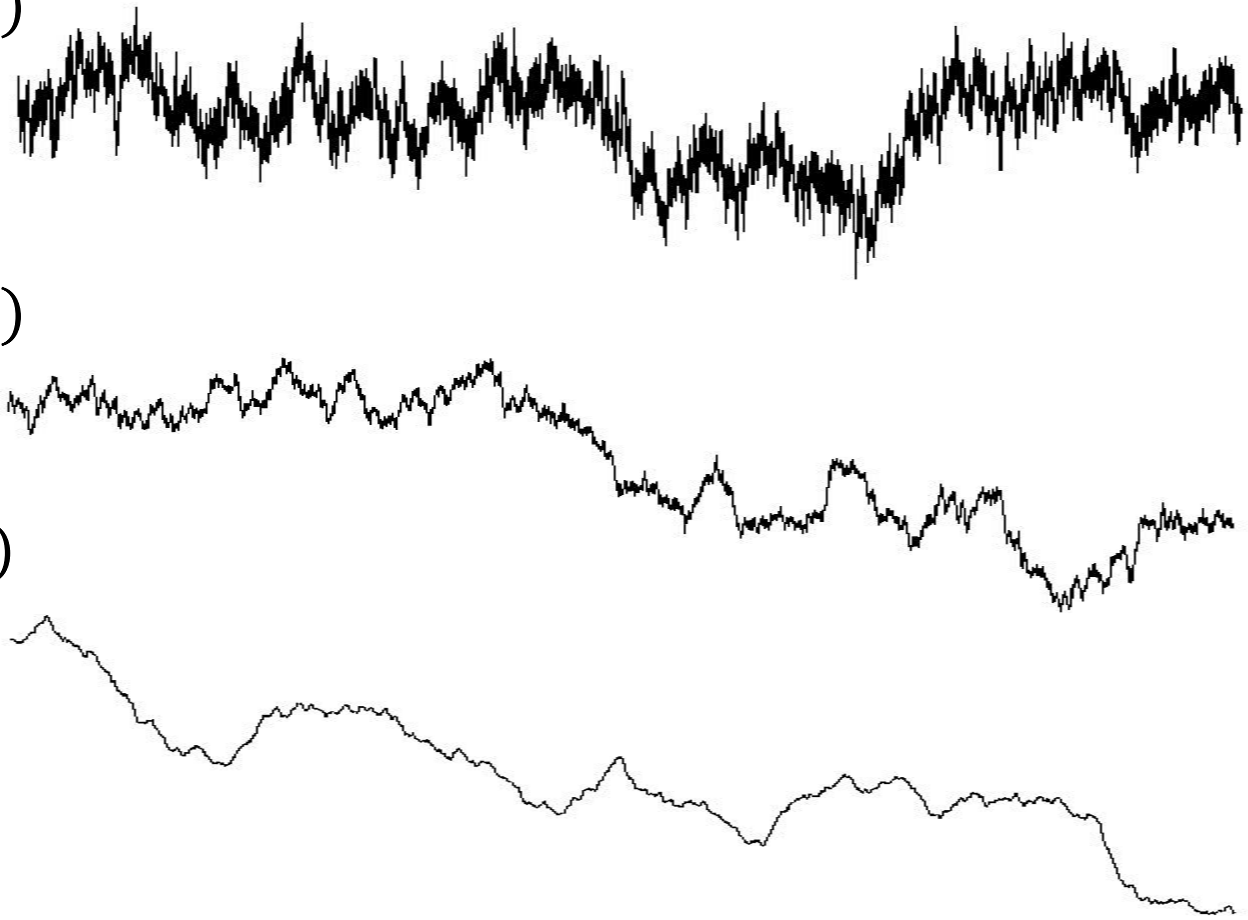
$d_f = 1.5$

$H \in (1/2,1)$

Persistent behaviour

$B_{0.8}(t)$

$d_f = 1.2$

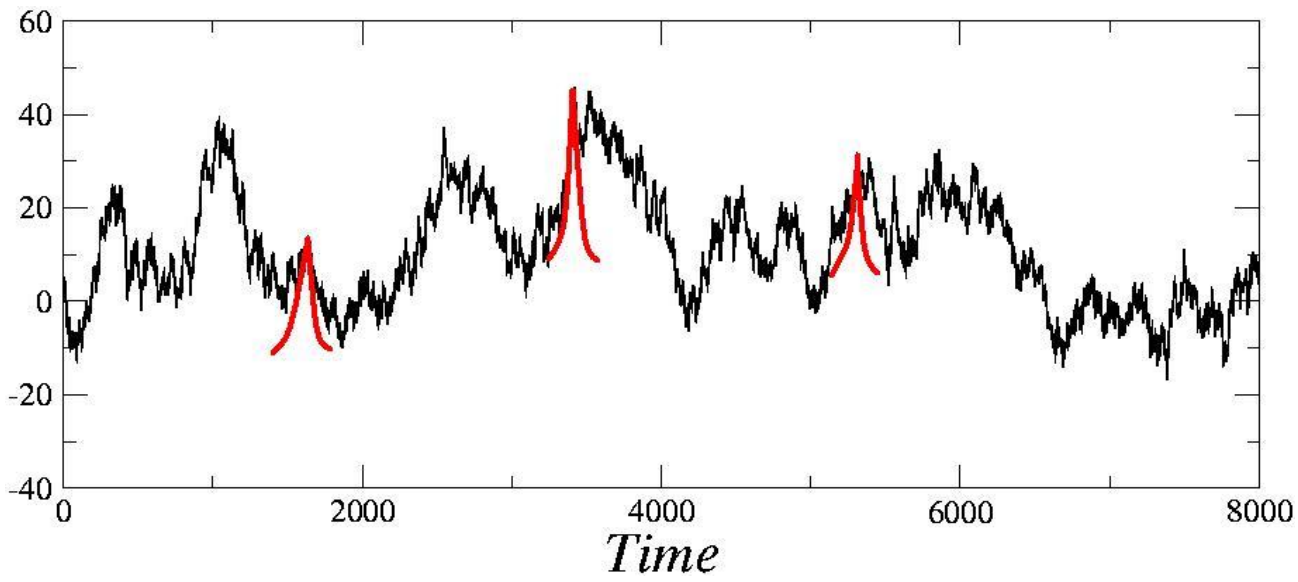


Sample-paths are almost nowhere differentiable.

Local Regularity of Functions

Local singular behaviour of f :

$$f(x) = c_0 + c_1(x - x_0) + \cdots + c_n(x - x_0)^n + C|x - x_0|^{\alpha(x_0)}$$



$\alpha(x_0)$ – Hölder exponent

- $\alpha(x_0) \nearrow$ – more regular function
- $\alpha(x_0) \searrow$ – less regular function

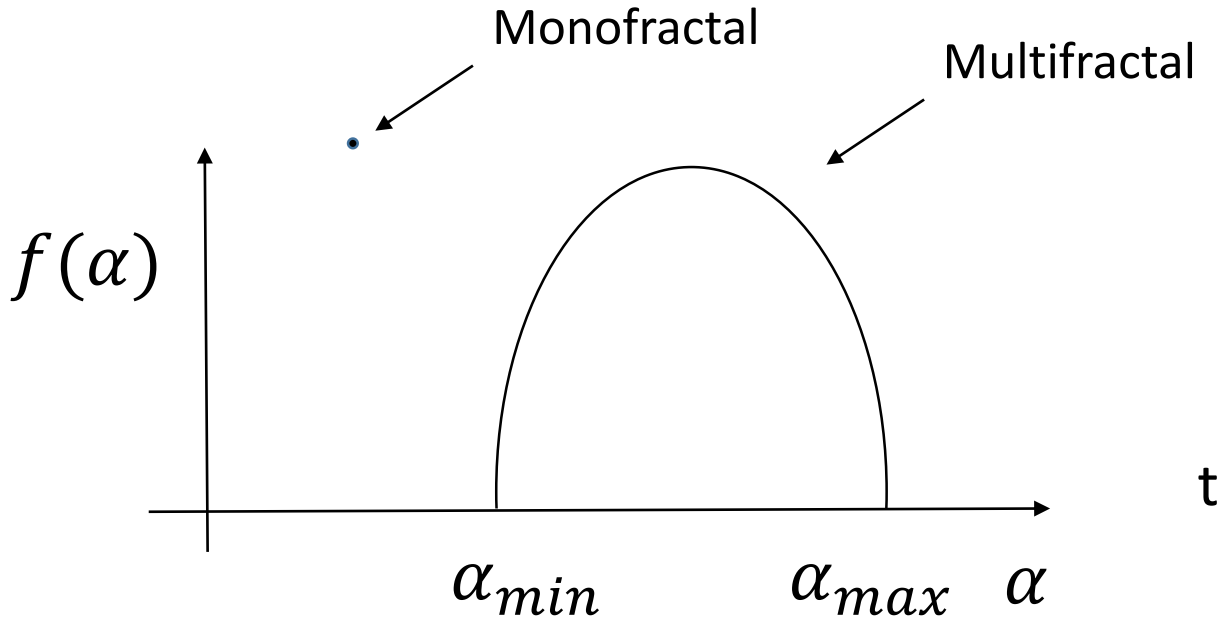
$$\alpha(x_0) = H$$

for fractional Brownian motion

Multifractal Spectrum

α – Hölder exponent

$$f(\alpha) = d_f(x, \alpha(x) = \alpha)$$



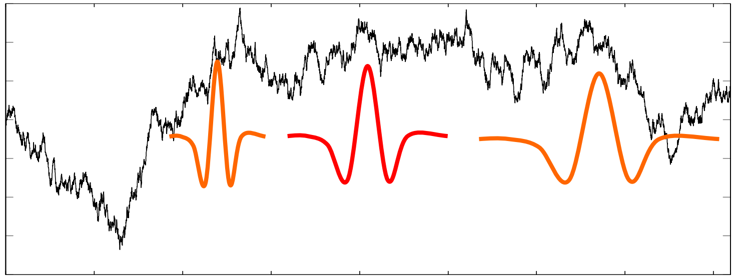
Width of the spectrum:

$$\Delta\alpha = \alpha_{max} - \alpha_{min}$$

The wider is spectrum
the more complex is the time series

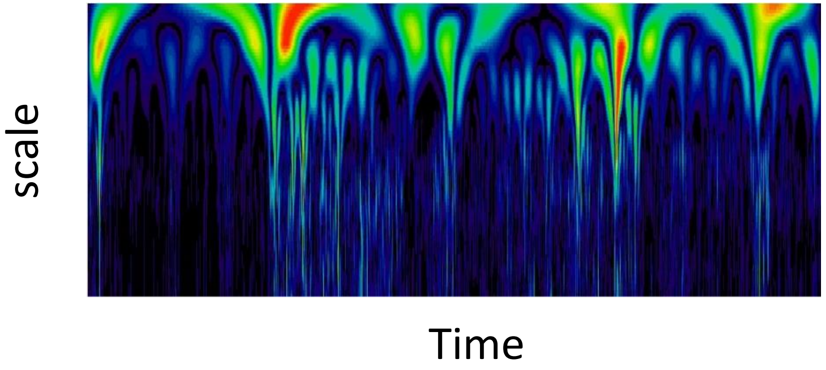
Wavelet Transform Modulus Maxima (WTMM)

x_i - time series

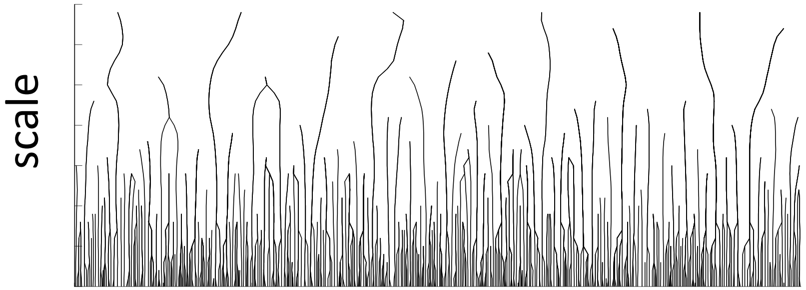


s – scale
n – time
ψ - wavelet

$$T_\psi(n,s) = \frac{1}{s}\sum_{i=1}^{N} X_i \psi[(i-n)/s]$$



Identifying positions of the local maxima T_Ψ



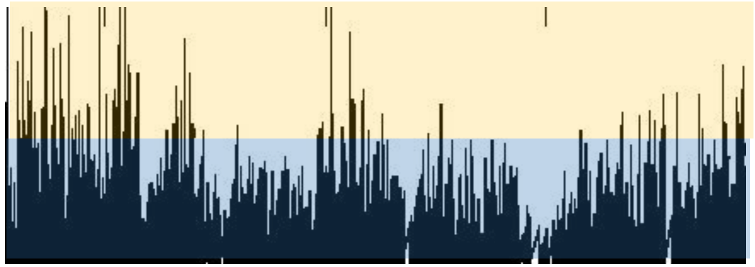
Calculating the partition function $Z(q,s)$

$$Z(q,s) = \sum_{l \in L(s')} \left| T_\psi(n_l(s), s) \right|^q$$

$$Z(q,s) \sim (s)^{\tau(q)}$$

$$\alpha = \tau'(q), \qquad f(\alpha) = q\alpha - \tau(q)$$

Multifractal detrended fluctuation analysis (MFDFA)



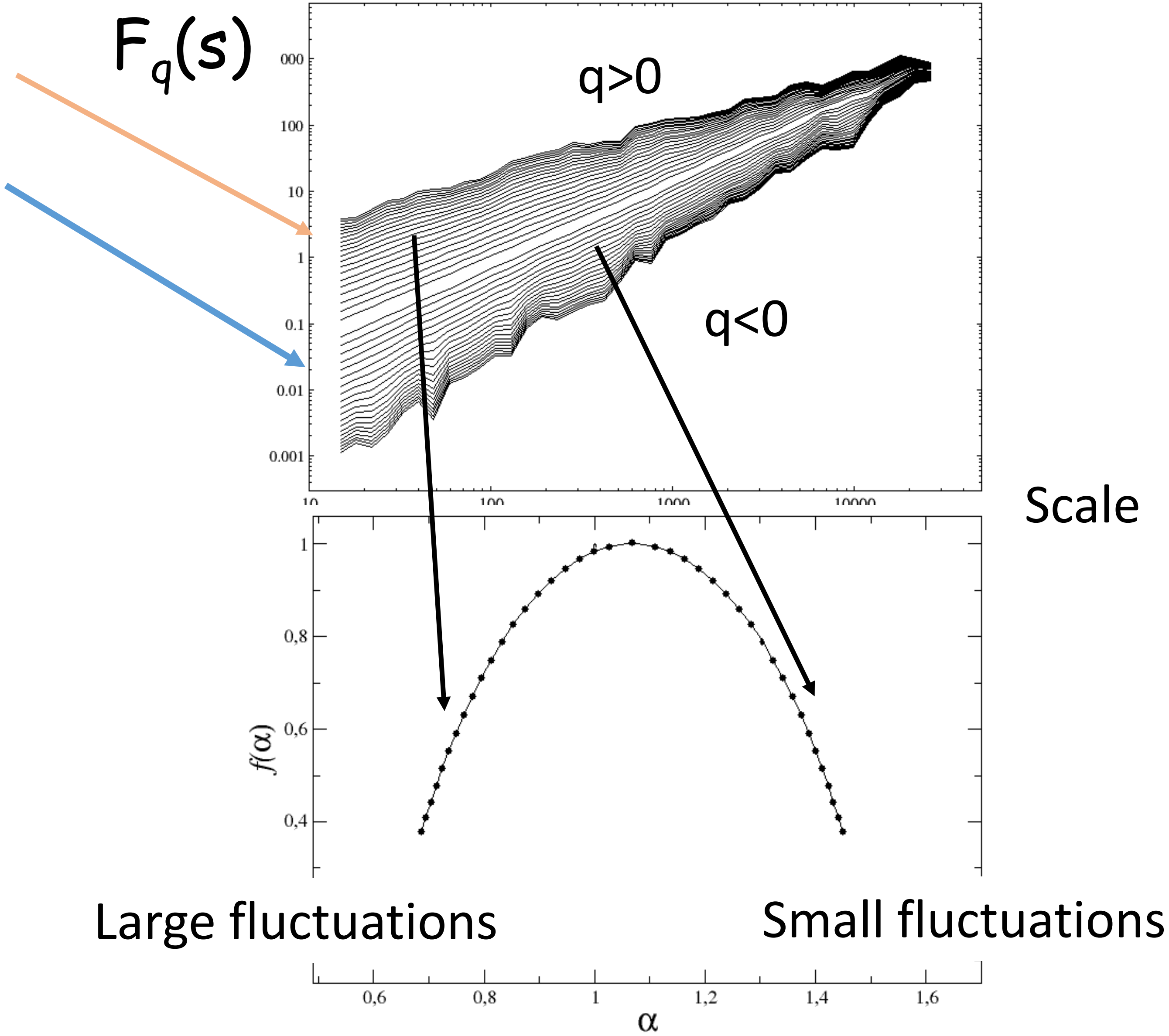
$$F^2(s,v) \equiv \frac{1}{s}\sum_{i=1}^{s}\{Y[(v-1)s+i]-y_v(i)\}^2$$

$$F_q(s) \equiv \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[F^2(s,v)\right]^{q/2}\right\}^{1/q}$$

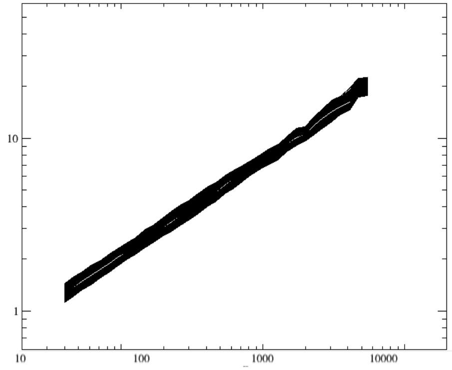
$$\boxed{F_q \sim s^{h(q)}}$$

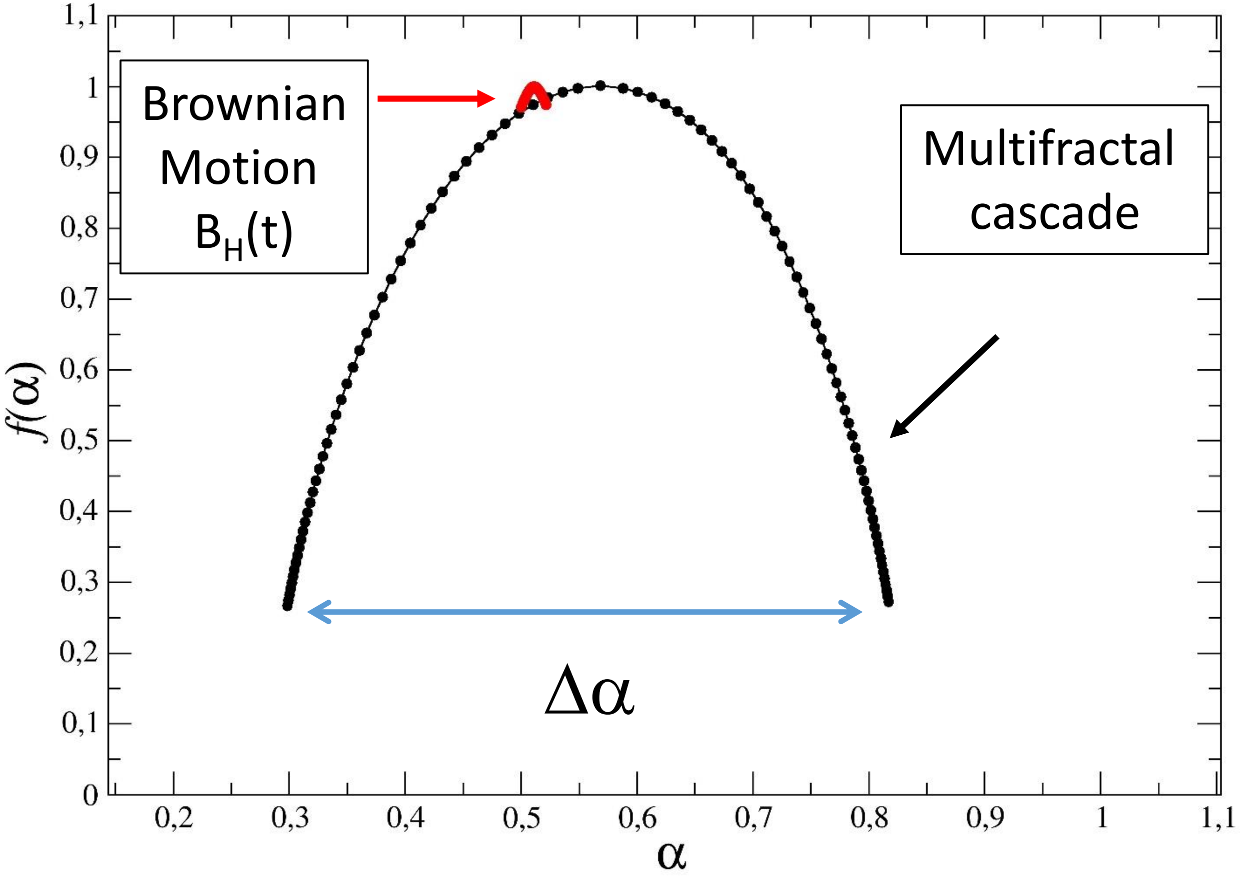
$$\tau(q) = qh(q) - 1$$

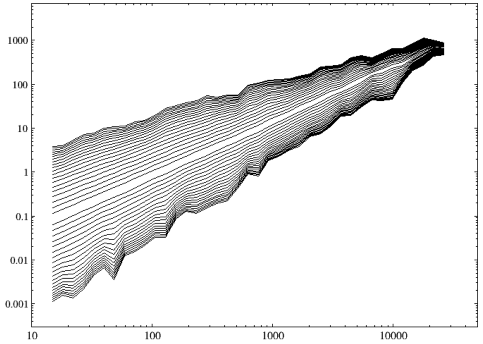
$$\alpha = \tau'(q), \qquad f(\alpha) = q\alpha - \tau(q)$$



Multifractal Spectrum
as a measure of complexity

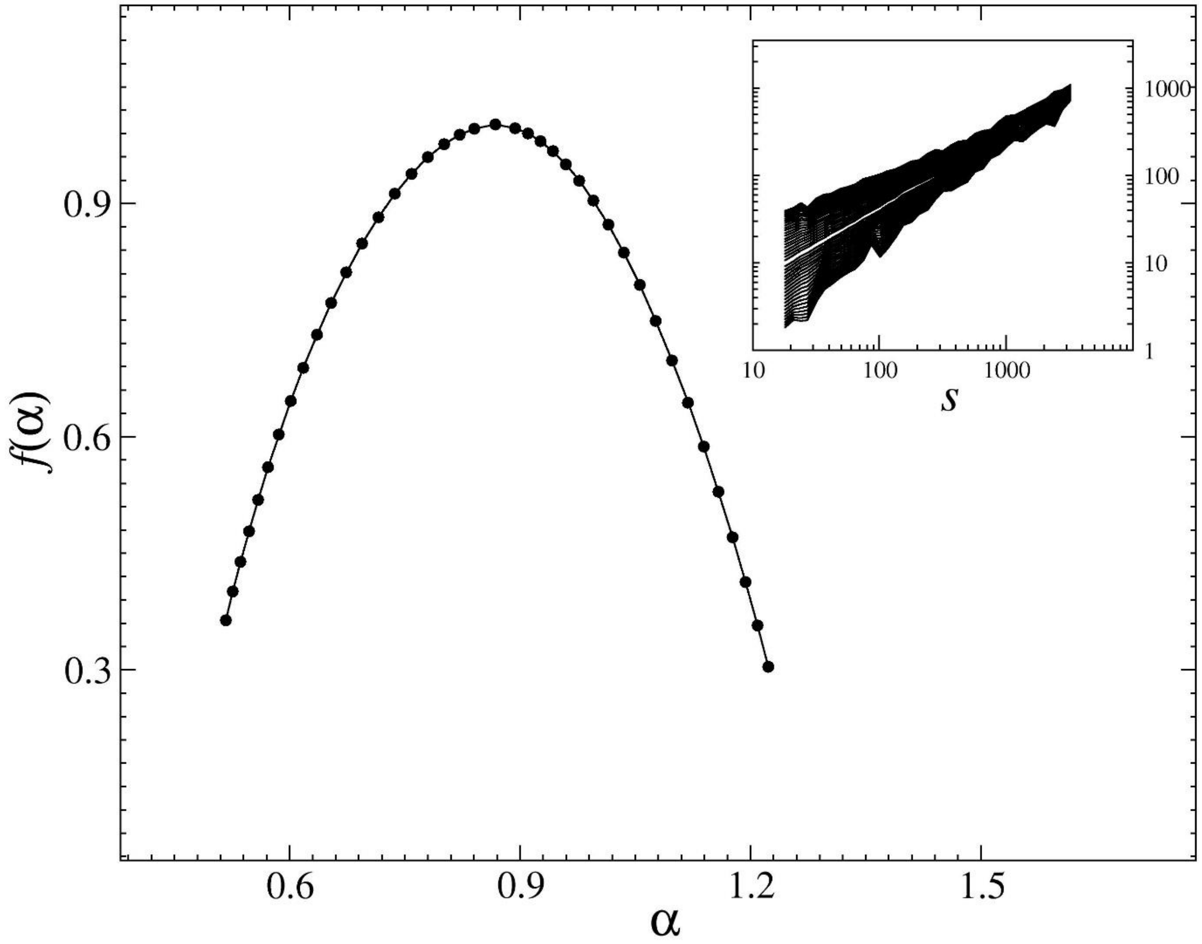






Finnegans Wake

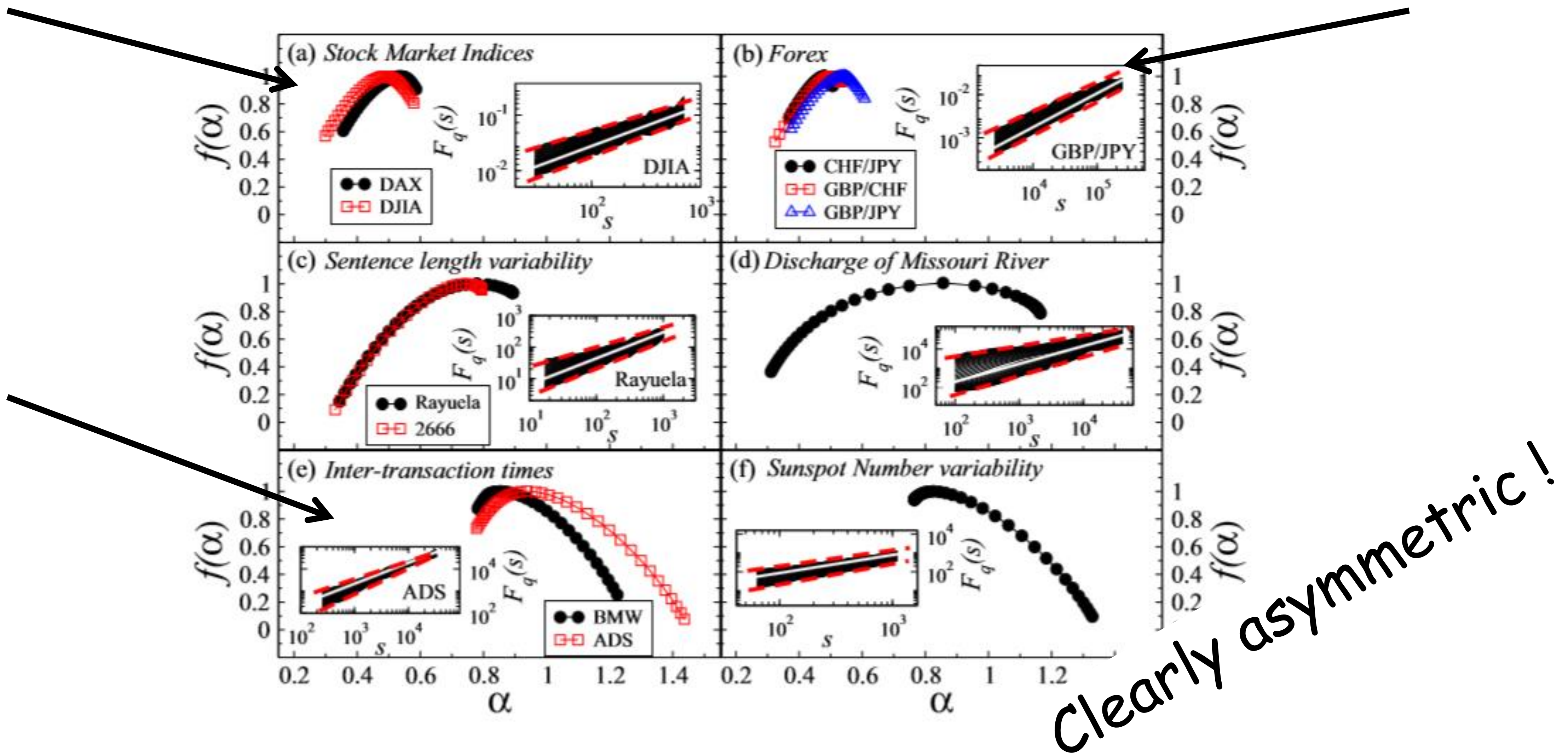
Sentence length variability



S. Drożdż, P. Oświęcimka, A. Kulig, J. Kwapień, K. Bazarnik, I. Grabska-Gradzińska, J. Rybicki, M. Stanuszek
Quantifying origin and character of long-range correlations in narrative texts,
Information Sciences 331, 32 (2016)

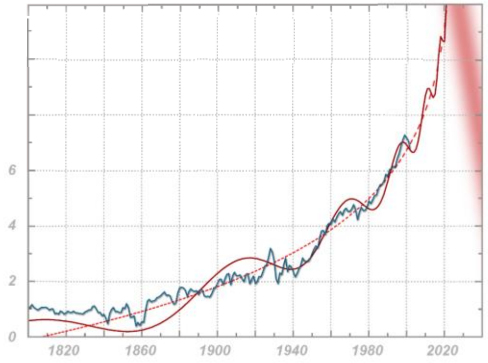
„Typical” Mutlifractal Characteristics

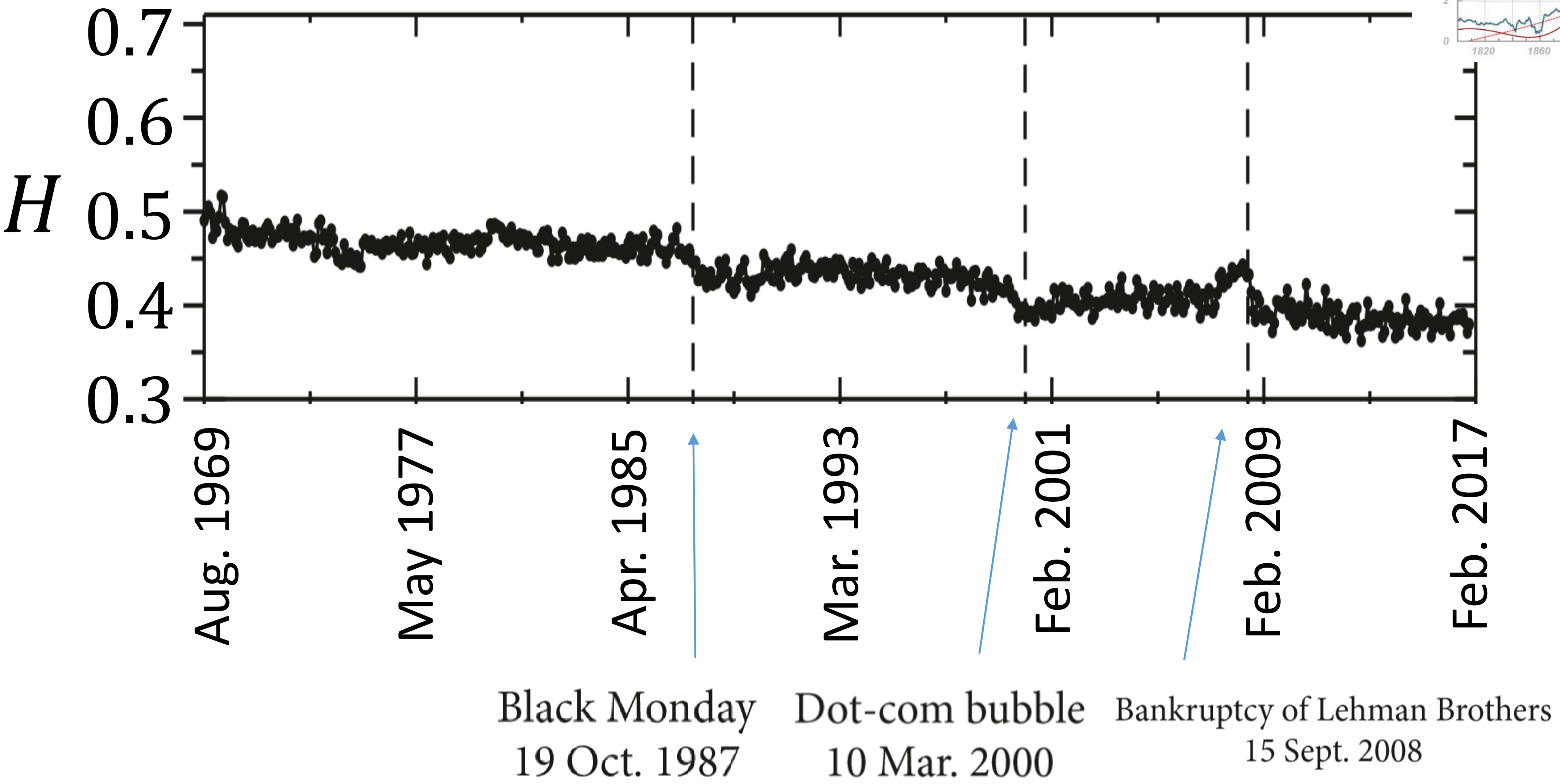
RAPID COMMUNICATIONS



The local Hurst exponent of the financial time series

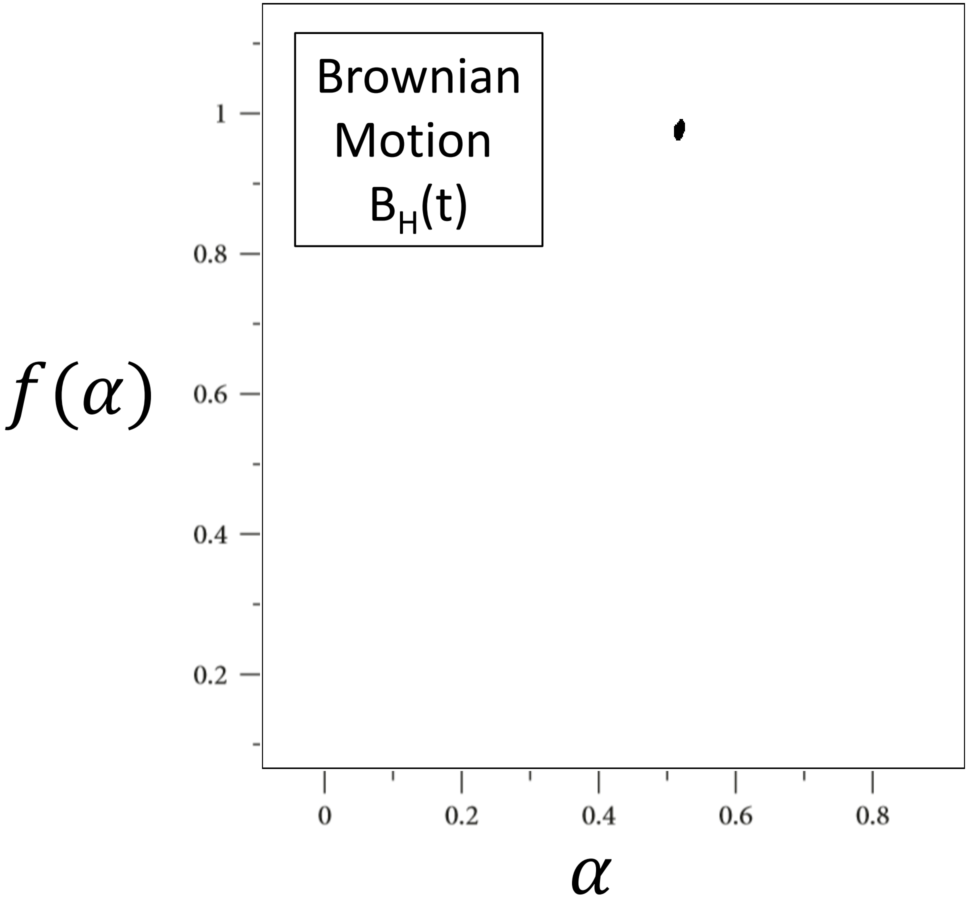
Daily prices of the S&P500 index
January, 1950 –December, 2016 (16,496datapoints).

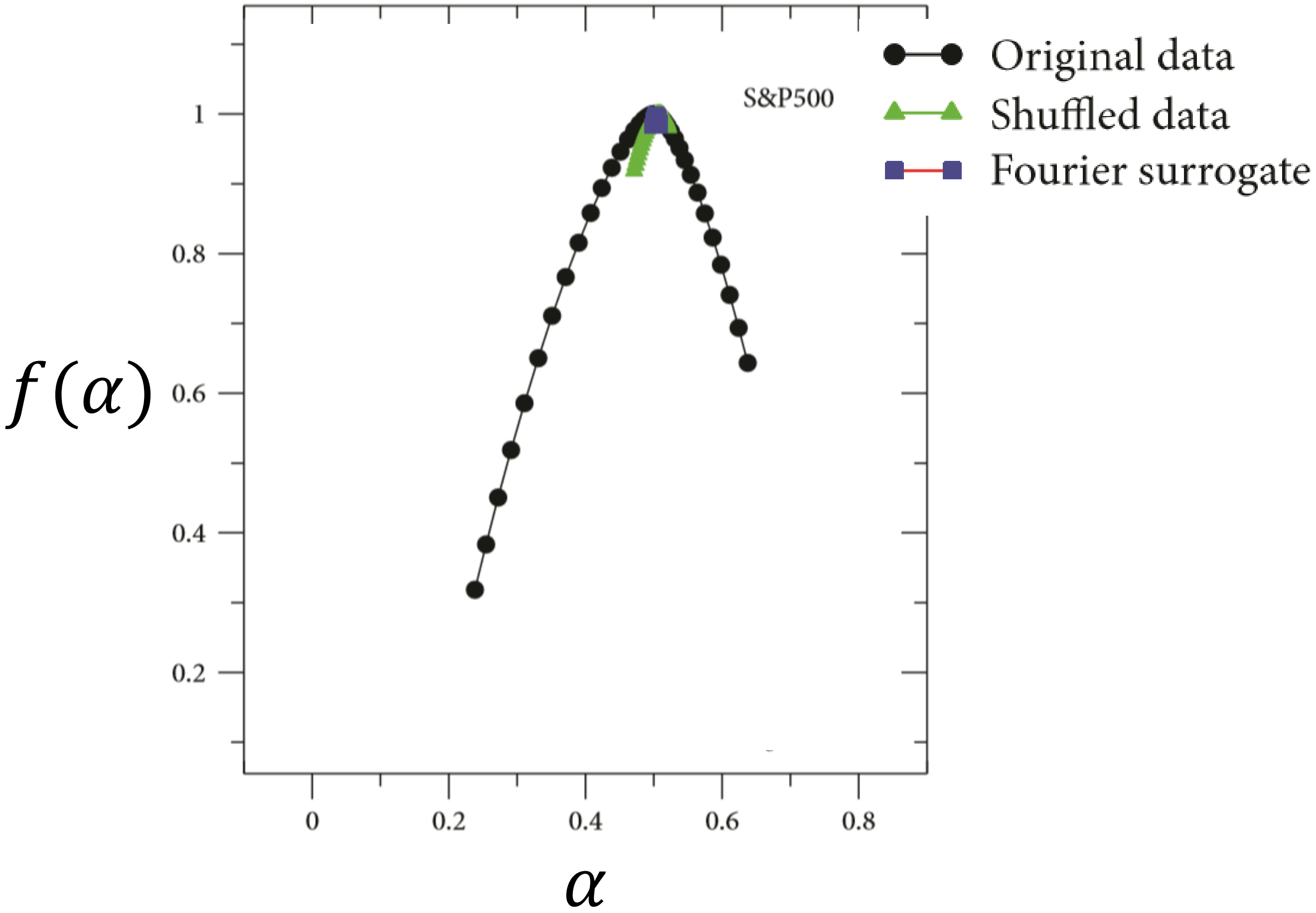




Multifractality of stock market

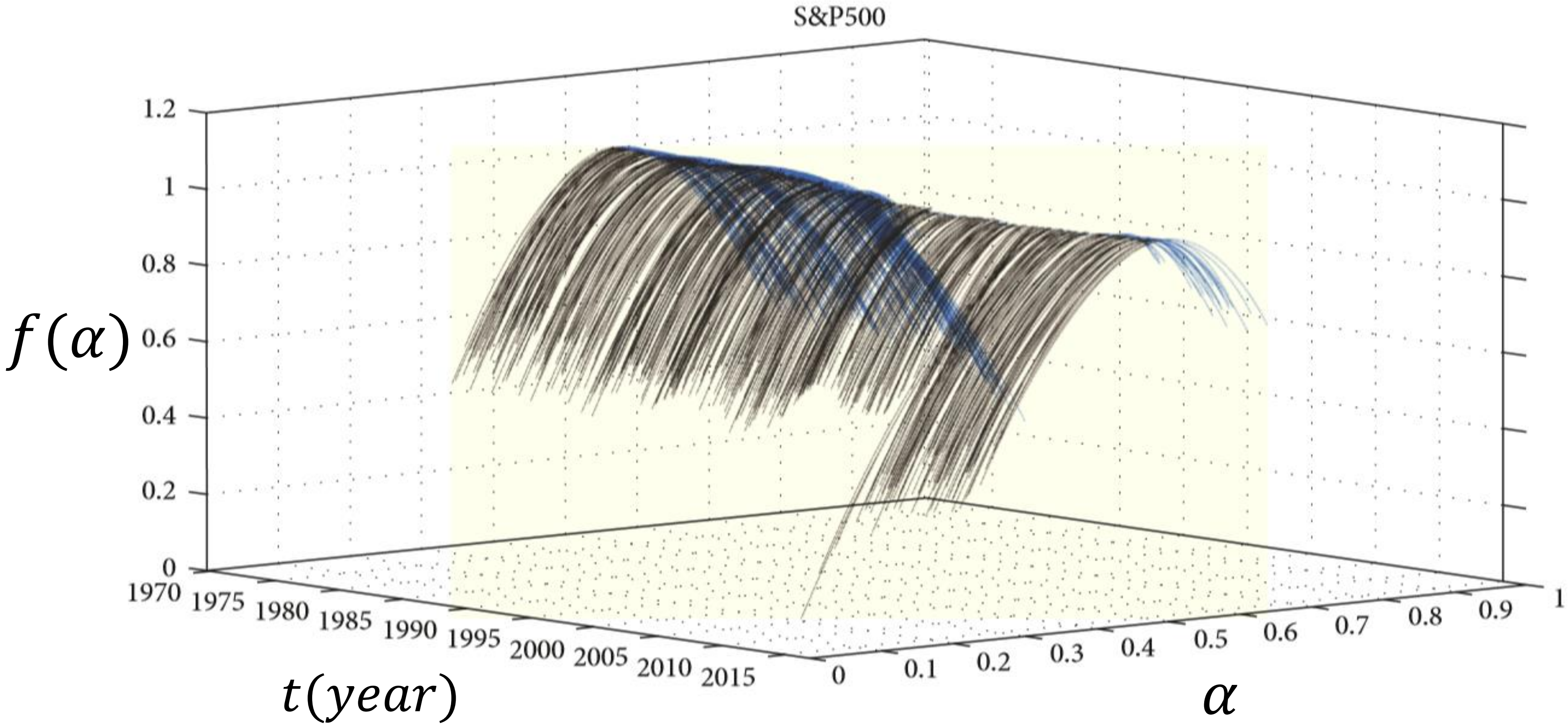
Daily prices of the S&P500 index
January, 1950 –December, 2016 (16,496datapoints).





S&P500 analysis

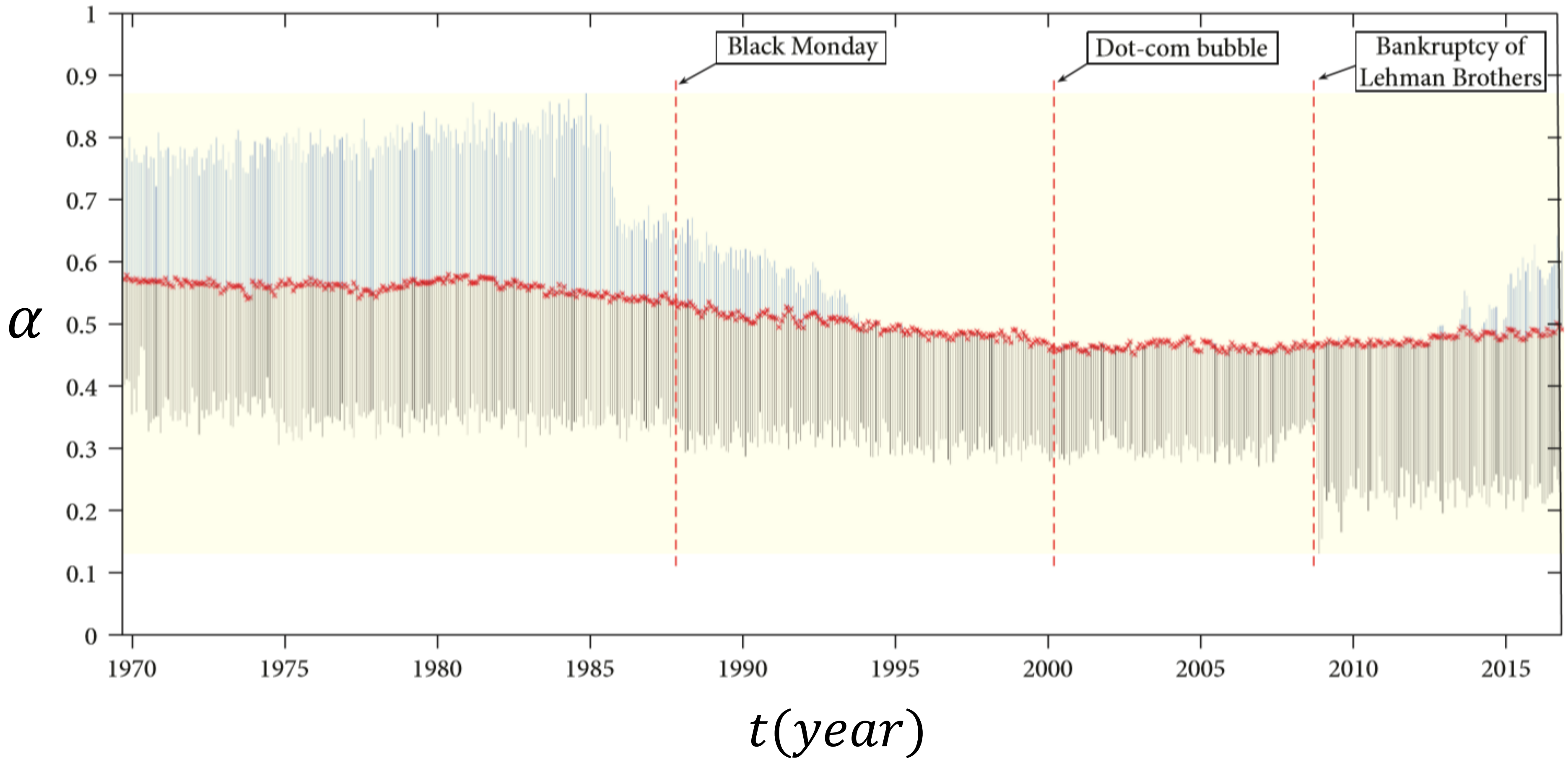
Singularity spectra $f(\alpha)$ calculated within a rolling 20-year window



S. Drożdż, R. Kowalski, P. Oświęcimka, R. Rak, R. Gębarowski, *Dynamical Variety of Shapes in Financial Multifractality,* Complexity, **2018**, Article ID 7015721

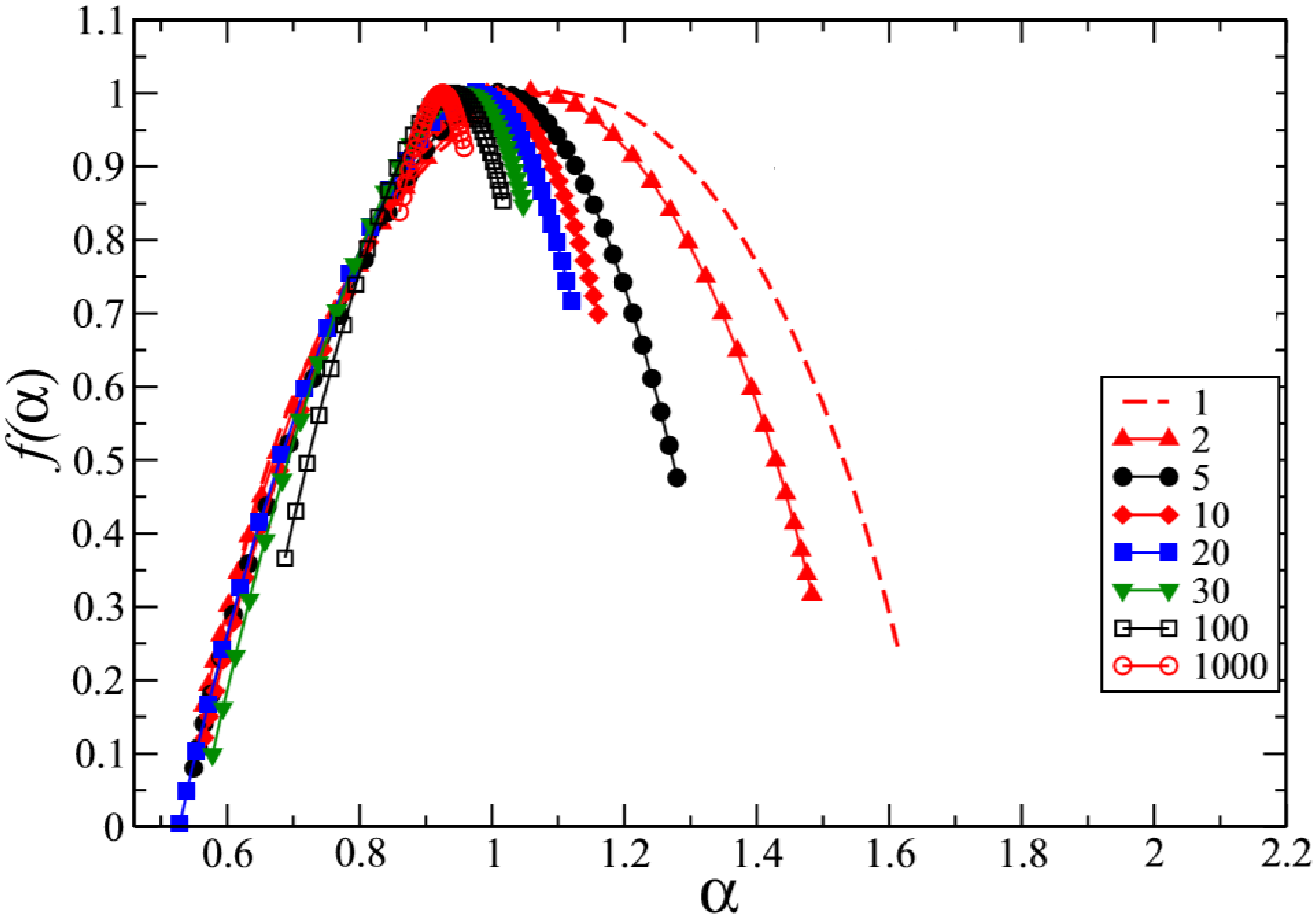
S&P500 analysis

Projections of f α of $f(\alpha)$ onto the time $t - \alpha$ plane



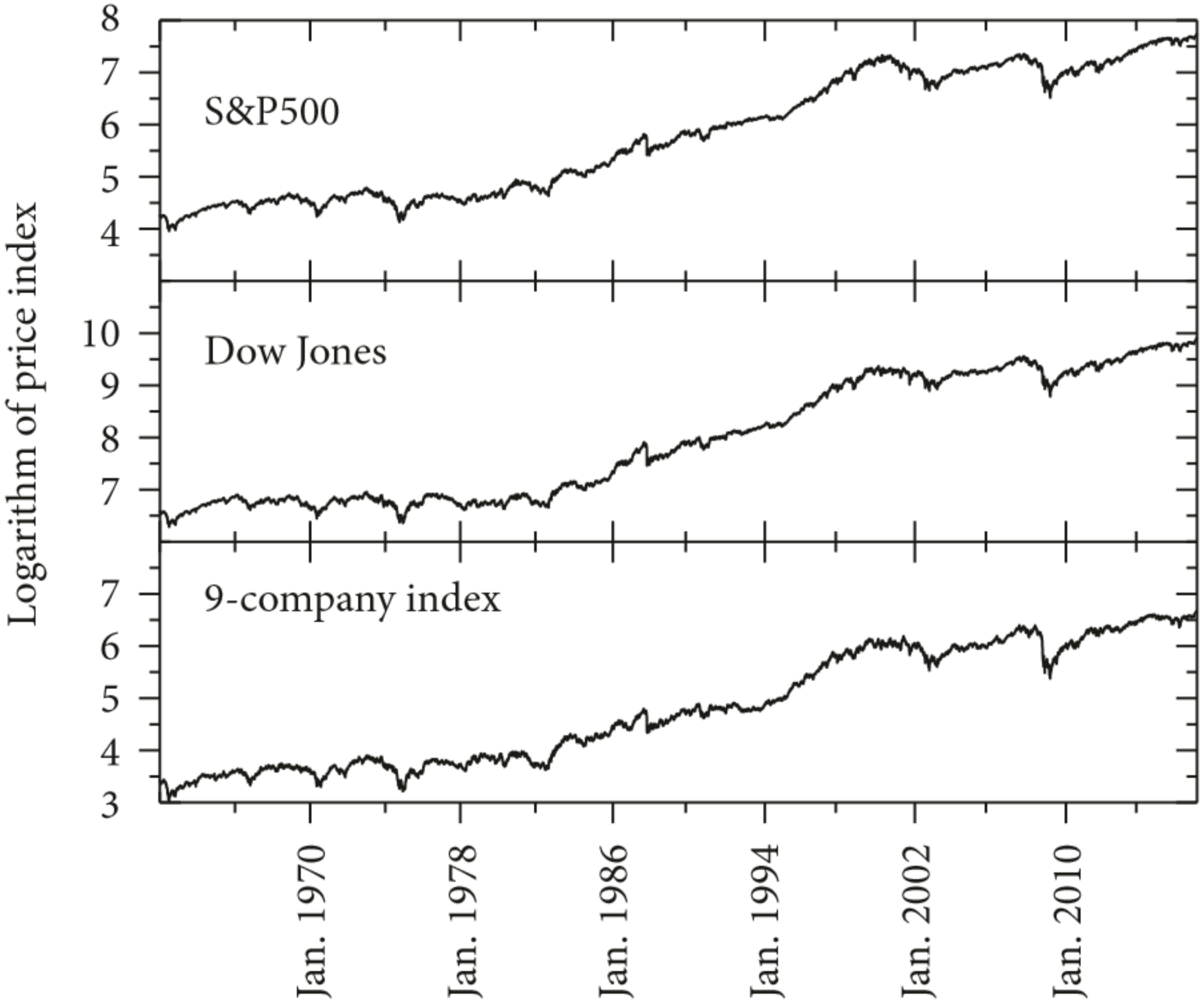
S. Drożdż, R. Kowalski, P. Oświęcimka, R. Rak, R. Gębarowski, *Dynamical Variety of Shapes in Financial Multifractality,* Complexity, **2018**, Article ID 7015721

Multifractal spectra for an increasing number of the superimposed binomial cascades



S. Drożdż, P. Oświęcimka, *Detecting and interpreting distortions in hierarchical organization of complex time series*, Physical Review E **91**, 030902(R) (2015)

S&P500, Dow Jones, and of the sum of 9 DJIA stocks

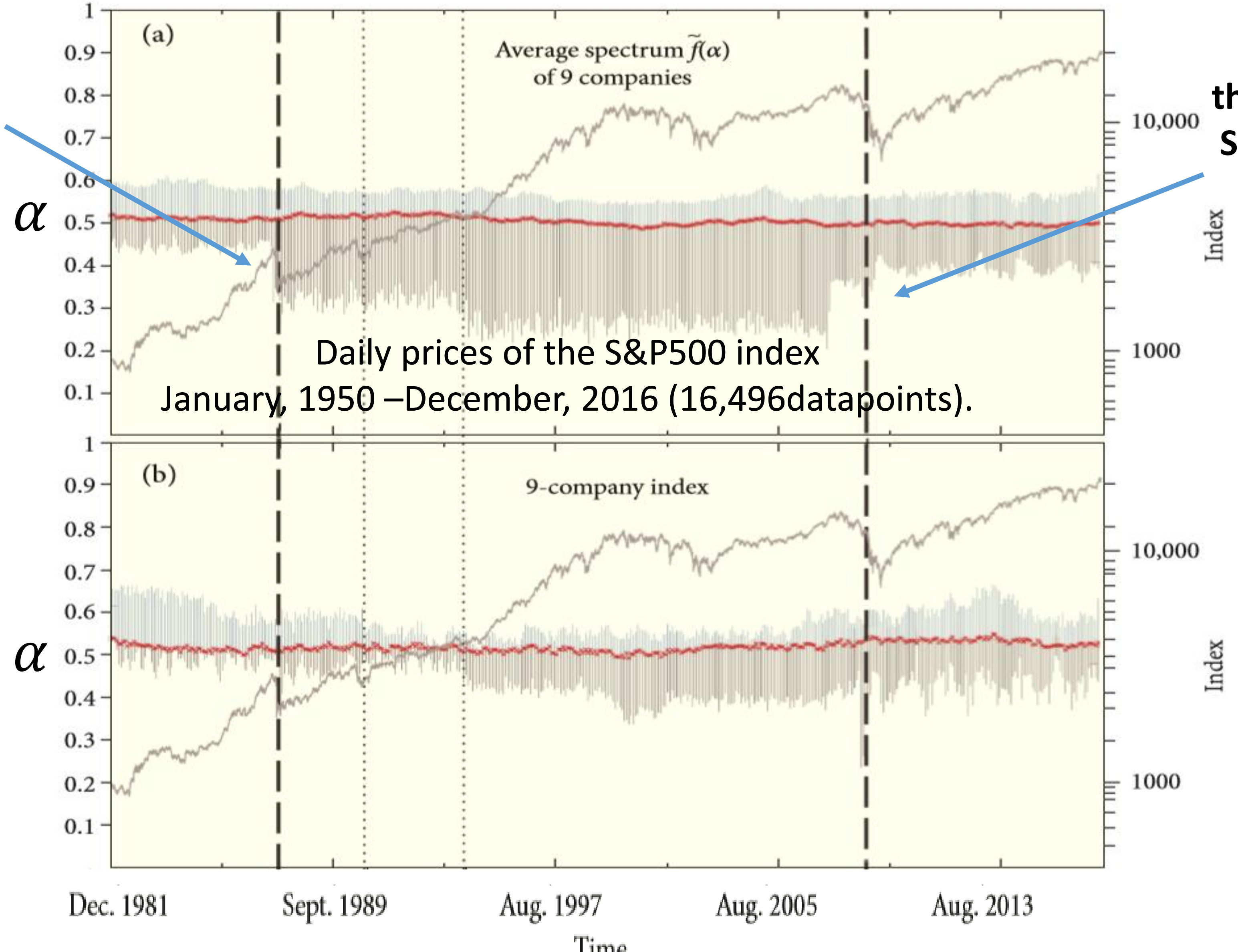


GE (General Electric),
AA (Alcoa),
IBM (International Business Machines),
KO (Coca-Cola),
BA (Boeing),
CAT (Caterpillar),
DIS (Walt Disney),
HPQ (Hewlett-Packard),
DD (DuPont)

Projections onto the time $t - \alpha$ plane of the sequence of singularity spectra $f(\alpha)$ calculated within a rolling 20-year window

Black Monday
October 19, 1987

Bankruptcy of
the Lehman Brothers
September 15, 2008



Thank you for your attention.